2

Sets and Counting

2.4 Permutations and Combinations

Objectives

- Develop and apply the Permutation Formula
- Develop and apply the Combination Formula
- Determine the number of distinguishable permutations

Permutations and Combinations

The Fundamental Principle of Counting allows us to determine the total number of possible outcomes when a series of decisions must be made.

If more than one item is selected, the selections can be made either *with* or *without* replacement.

With versus Without Replacement

Selecting items *with replacement* means that the same item *can* be selected more than once; after a specific item has been chosen, it is put back into the pool of future choices.

With versus Without Replacement

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Selecting items *without replacement* means that the same item *cannot* be selected more than once; after a specific item has been chosen, it is not replaced.

With versus Without Replacement

Example:

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Pin: 3663 or 7987

The total number of outcomes:

10	10	10	10

$10^4 = 10{,}000$

Answer: 10,000 possible four-digit PINs.

With versus Without Replacement

In many situations, items cannot be selected more than once.

For instance, when selecting a committee of three people from a group of twenty, you cannot select the same person more than once.

Once you have selected a specific person (say, Lauren), you do not put her back into the pool of choices.

With versus Without Replacement

When selecting items without replacement, depending on whether the order of selection is important, *permutations* or *combinations* are used to determine the total number of possible outcomes.

Permutations

Permutations

When more than one item is selected (without replacement) from a single category, and the order of selection *is* important, the various possible outcomes are called **permutations**.

Example 1 – *Finding the Number of Permutations*

Six local bands have volunteered to perform at a benefit concert, but there is enough time for only four bands to play. There is also some concern over the order in which the chosen bands will perform. How many different lineups are possible?

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- the bands are selected without replacement;

(a band cannot be selected to play and then be re-selected to play again)

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Six local bands have volunteered to perform at a benefit concert, but there is enough time for only four bands to play. There is also some concern over the order in which the chosen bands will perform. How many different lineups are possible?

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The computation in Example 1 is similar to a factorial, but the factors do not go all the way down to 1; the product $6 \cdot 5 \cdot 4 \cdot 3$ is a “truncated” (cut-off) factorial.

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We can change this truncated factorial into a complete factorial in the following manner :

$$6 \cdot 5 \cdot 4 \cdot 3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot (2 \cdot 1)}{(2 \cdot 1)} \qquad \text{multiplying by } \tfrac{2}{2} \text{ and } \tfrac{1}{1}$$

$$= \frac{6!}{2!}$$

Permutations

Notice that this last expression can be written as

$$\frac{6!}{2!} = \frac{6!}{(6 - 4)!}.$$

This result is generalized as follows.

Permutation Formula

The number of **permutations**, or arrangements, of r items selected without replacement from a pool of n items ($r \leq n$), denoted by ${}_nP_r$, is

$$P(n,r) = {}_nP_r = \frac{n!}{(n - r)!}$$

Permutations are used whenever more than one item is selected (without replacement) from a category and the order of selection is important.

Permutations

referring to Example 1: 4 bands selected from a pool of 6 can be denoted by

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Other notations can be used to represent the number of permutations of a group of items:

${}_nP_r$ $P(n, r)$ P_r^n and $P_{n,\,r}$

all represent the number of possible permutations (or arrangements) of *r* items selected (without replacement) from a pool of *n* items.

Combinations

Combinations

When items are selected from a group, the order of selection may or may not be important.

When more than one item is selected (without replacement) from a single category and the order of selection is not important, the various possible outcomes are called **combinations**.

Example 4 – *Listing All Possible Combinations*

Two adults are needed to chaperone a daycare center's field trip. Marcus, Vivian, Frank, and Keiko are the four managers of the center. How many different groups of chaperones are possible?

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Solution:

In selecting the chaperones, the order of selection is *not* important; "Marcus and Vivian" is the same as "Vivian and Marcus."

Hence, the permutation formula cannot be used.

Example 4 – *Solution*

cont'd

We do not yet have a shortcut for finding the total number of possibilities when the order of selection is not important, hence we must list all the possibilities:

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Marcus and Vivian	Marcus and Frank	Marcus and Keiko
Vivian and Frank	Vivian and Keiko	Frank and Keiko

Answer: 6 different groups of 2 chaperones are possible from the group of 4 managers

Combinations

Just as ${}_nP_r$ denotes the number of *permutations* of r elements selected from a pool of n elements, ${}_nC_r$ denotes the number of *combinations* of r elements selected from a pool of n elements.

In Example 4, we found that there are 6 combinations of 2 people selected from a pool of 4 by listing all six of the combinations, that is, ${}_4C_2 = 6$.

If we had a larger pool, listing each combination to find out how many there are would be extremely time consuming and tedious!

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If we had a larger pool, listing each combination to find out how many there are would be extremely time consuming and tedious!

Combinations

Combination Formula

The number of distinct **combinations** of r items selected without replacement from a pool of n items ($r \leq n$), denoted by ${}_nC_r$, is

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

Combinations are used whenever one or more items are selected (without replacement) from a category and the order of selection is not important.

Other notations can be used to represent the number of permutations of a group of items:

${}_nC_r$ $C(n, r)$ C_r^n and $C_{n,\,r}$

Example 5 – *Finding the Number of Combinations*

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Solution:

Because the order of selection is *not* important, we find the number of combinations when five items are selected from a pool of fifty:

$$C(50,5) = {}_{50}C_5 = \frac{50!}{(50-5)!\,5!}$$

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

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Solution:

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$$C(50,5) = {}_{50}C_5 = \frac{50!}{(50-5)!} = \frac{50!}{45!\,5!}$$

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

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$$C(50,5) = {}_{50}C_5 = \frac{50!}{(50-5)!} = \frac{50!}{45!\,5!}$$

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

$$= \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2{,}118{,}760$$

Example 7 – *Evaluating the Combination Formula*

$$C(n,r)=\frac{n!}{(n-r)!\,r!}$$

Find the value of ${}_5C_r$ for the following values of r :

a. $r = 0$

b. $r = 1$

c. $r = 2$

d. $r = 3$

e. $r = 4$

f. $r = 5$

Example 7 – *Evaluating the Combination Formula*

$C(n,r)=\frac{n!}{(n-r)!r!}$

Find the value of ${}_5C_r$ for the following values of *r* :

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$$\textbf{a.}\ {}_5C_0 = \frac{5!}{(5-0)!\cdot 0!} = \frac{5!}{5!\cdot 0!} = 1$$

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$$\textbf{d.}\quad {}_5C_3 = \frac{5!}{(5-3)!\cdot 3!} = \frac{5!}{2!\cdot 3!} = 10$$

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e. $r = 4$

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Find the value of ${}_5C_r$ for the following values of r :

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e. $r = 4$

$$\textbf{e.}\quad {}_5C_4 = \frac{5!}{(5-4)!\cdot 4!} = \frac{5!}{1!\cdot 4!} = 5$$

f. $r = 5$

$$\textbf{f.}\quad {}_5C_5 = \frac{5!}{(5-5)!\cdot 5!} = \frac{5!}{0!\cdot 5!} = 1$$

Combinations

The combinations generated in Example 7 exhibit a curious pattern. Notice that the values of ${}_5C_r$ are symmetric:

${}_5C_0 = {}_5C_5$, ${}_5C_1 = {}_5C_4$, and ${}_5C_2 = {}_5C_3$.

Combinations

Now examine the diagram in Figure 2.41. Each number in this “triangle” of numbers is the sum of two numbers in the row immediately above it. For example, 2 = 1 + 1 and 10 = 4 + 6, as shown by the inserted arrows.

1st row						1					
2nd row					1		1				
3rd row				1		2		1			
4th row			1		3		3		1		
5th row		1		4		6		4		1	
6th row	1		5		10		10		5		1
and so on											

Pascal’s triangle.

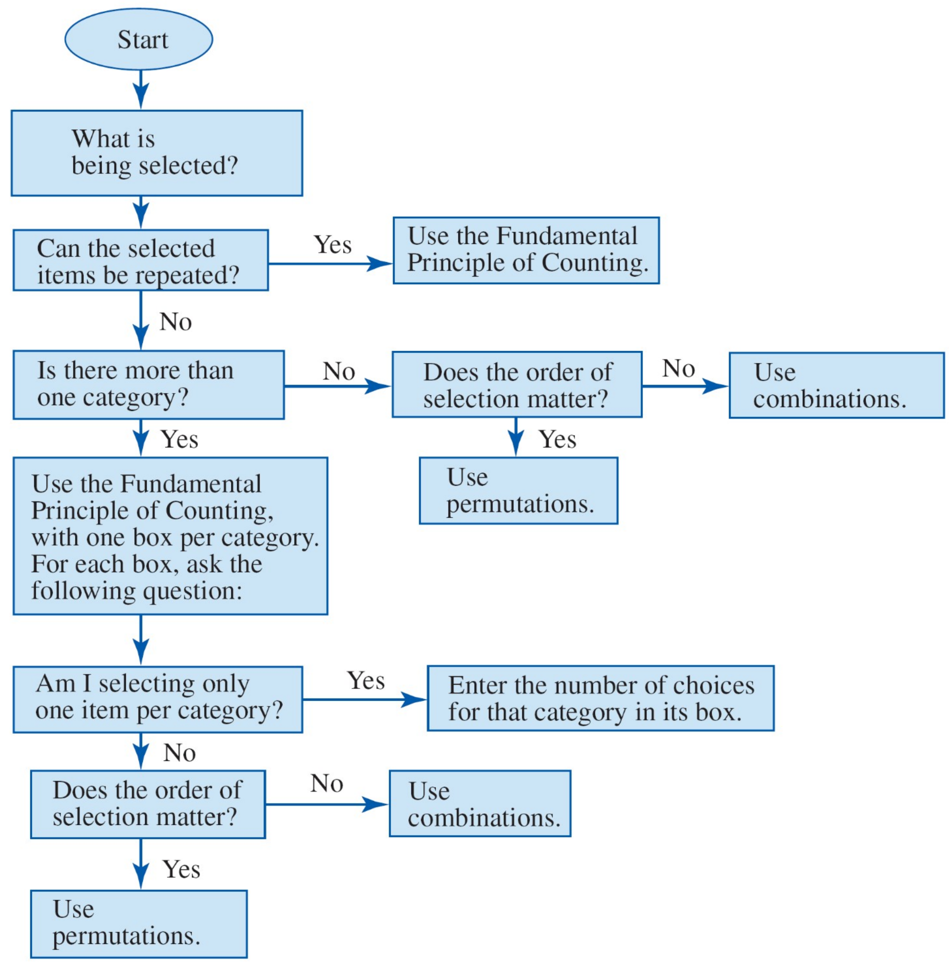
Figure 2.41

Combinations

Combinations

The most important part of any problem involving combinatorics is deciding which counting technique (or techniques) to use.

The following list of general steps and the flowchart in can help you to decide which method or methods to use in a specific problem.



Which counting technique?

Figure 2.42

Combinations

Which Counting Technique?

1. What is being selected?
2. If the selected items can be repeated, use the **Fundamental Principle of Counting** and multiply the number of choices for each category.
3. If there is only one category, use
   combinations if the order of selection does not matter—that is, r items can be selected from a pool of n items in ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$ ways.
   permutations if the order of selection does matter—that is, r items can be selected from a pool of n items in ${}_nP_r = \frac{n!}{(n-r)!}$ ways.
4. If there is more than one category, use the **Fundamental Principle of Counting** with one box per category.
   a. If you are selecting one item per category, the number in the box for that category is the number of choices for that category.
   b. If you are selecting more than one item per category, the number in the box for that category is found by using step 3.

Permutations of Identical Items

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Permutations of Identical Items

In how many different ways can the three letters in the word "SAW" be arranged? As we know, arrangements are referred to as *permutations,* so we can apply the Permutation Formula, ${}_nP_r = \frac{n!}{(n - r)!}$.

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Therefore,

$$ {}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3 \cdot 2 \cdot 1}{1} = 6 $$

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Therefore,

$$ {}_3P_3 = \frac{3!}{(3 - 3)!} = \frac{3!}{0!} = \frac{3 \cdot 2 \cdot 1}{1} = 6 $$

The six permutations of the letters in SAW are

SAW SWA AWS ASW WAS WSA

Permutations of Identical Items

What happens if some of the items are the same (identical)? For example, in how many different ways can the three letters in the word “SEE” be arranged?

Permutations of Identical Items

What happens if some of the items are the same (identical)? For example, in how many different ways can the three letters in the word “SEE” be arranged?

Two of the letters are identical (E), hence we cannot use the Permutation Formula directly; we take a slightly different approach.

Permutations of Identical Items

Temporarily, let us assume that the E’s are written in different colored inks, say, red and blue.

Therefore, SEE could be expressed as SEE.

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These three symbols could be arranged in 6 ways as follows:

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These three symbols could be arranged in 6 ways as follows:

SEE   SEE   ESE   ESE   EES   EES

If we now remove the color, the arrangements are

SEE   SEE   ESE   ESE   EES   EES

Permutations of Identical Items

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These three symbols could be arranged in 6 ways as follows:

SEE SEE ESE ESE EES EES

If we now remove the color, the arrangements are

SEE SEE ESE ESE EES EES

There are 3 different ways can the three letters in the word "SEE" be arranged

Permutations of Identical Items

Notice that when n = 3 (the total number of letters in SEE) and x = 2 (the number of identical letters), we can divide n! by x! to obtain the number of distinguishable permutations; that is,

$$\frac{n!}{x!} = \frac{3!}{2!} = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 3$$

Permutations of Identical Items

This method is applicable because dividing by the factorial of the repeated letter eliminates the duplicate arrangements; the method may by generalized as follows.

Distinguishable Permutations of Identical Items

The number of **distinguishable permutations** (or arrangements) of n items in which x items are identical, y items are identical, z items are identical, and so on, is $\frac{n!}{x!y!z!\cdots}$. That is, to find the number of distinguishable permutations, divide the total factorial by the factorial of each repeated item.

Example 9 – *Finding the Number of Distinguishable Permutations*

Find the number of distinguishable permutations of the letters in the word "MISSISSIPPI."

Solution:

The word "MISSISSIPPI" has n = 11 letters;
I is repeated x = 4 times,

S is repeated y = 4 times,
and P is repeated z = 2 times.

Therefore, we get $$\frac{n!}{x!y!z!} = \frac{11!}{4!4!2!} = 34{,}650$$

The letters in the word MISSISSIPPI can be arranged in 34,650 ways.

Example 9 – *Solution*

cont'd

$$= 34{,}650$$