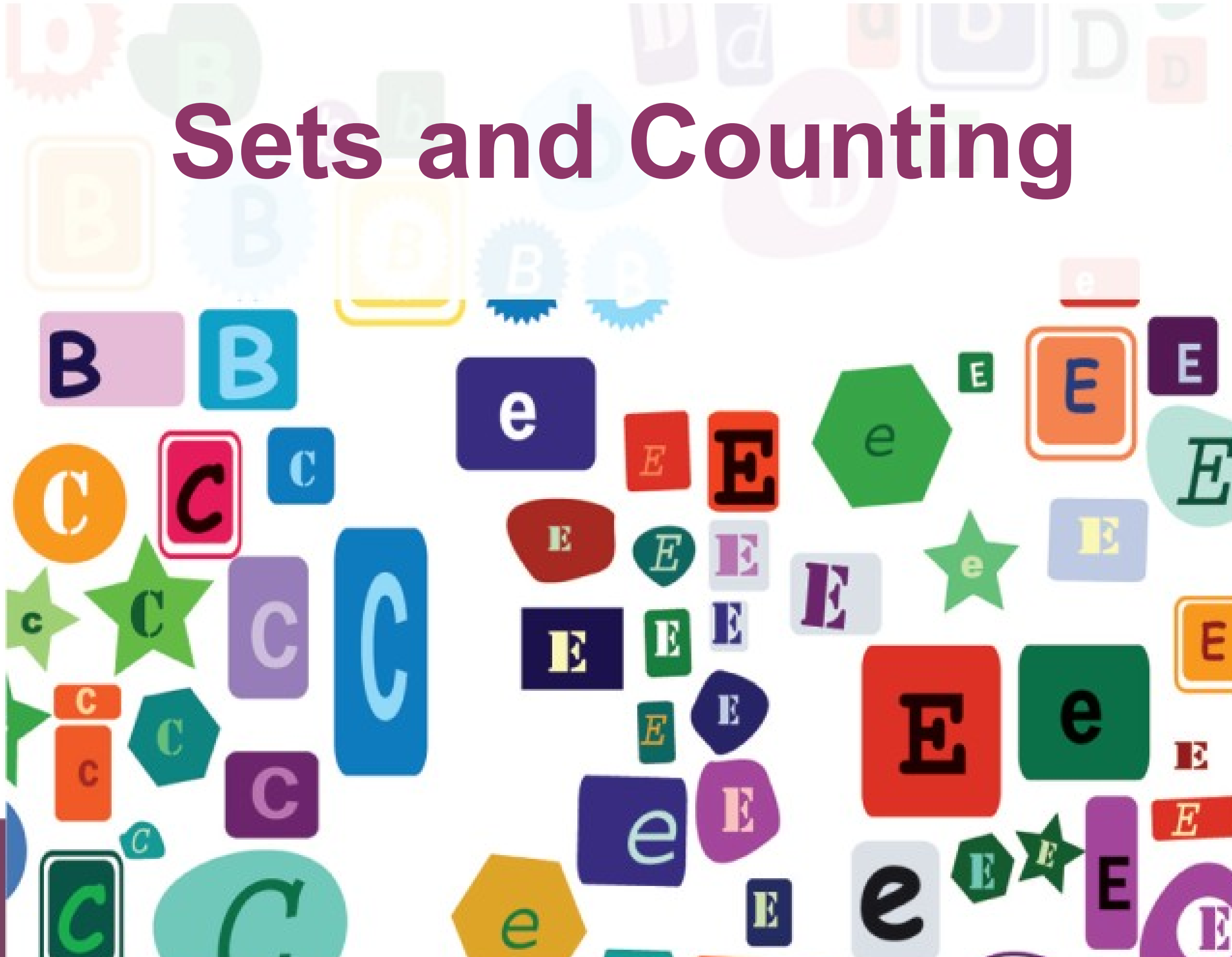
2

Sets and Counting



2.1 Sets and Set Operations

Objectives

- Learn the basic vocabulary and notation of set theory
- Learn and apply the union, intersection, and complement operations
- Draw Venn diagrams

Sets and Set Operations

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In geography, we can talk about the *set* of all state capitals or the *set* of all states west of the Mississippi. It is easy to determine whether something is in these sets; for example, Albany is an element of the set of state capitals, whereas New York City is not.

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In geography, we can talk about the *set* of all state capitals or the *set* of all states west of the Mississippi. It is easy to determine whether something is in these sets; for example, Albany is an element of the set of state capitals, whereas New York City is not.

Such sets are called **well defined** because there is a *way of determining for sure whether a particular item is an element of the set*.

Example 1 – *Determining Well-Defined Sets*

Which of the following sets are well-defined?

a. the set of all movies directed by Alfred Hitchcock

b. the set of all great rock-and-roll bands

c. the set of all possible two-person committees selected from a group of five people

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b. the set of all great rock-and-roll bands

c. the set of all possible two-person committees selected from a group of five people

Solution:

a. This set is well-defined; either a movie was directed by Hitchcock, or it was not.

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a. the set of all movies directed by Alfred Hitchcock

b. the set of all great rock-and-roll bands

c. the set of all possible two-person committees selected from a group of five people

Solution:

b. This set is *not* well-defined; membership is a matter of opinion. Some people would say that the Ramones (one of the pioneer punk bands of the late 1970s) are a member, whereas others might say they are not.

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a. the set of all movies directed by Alfred Hitchcock

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c. the set of all possible two-person committees selected from a group of five people

Solution:

c. This set is well-defined; either the two people are from the group of five, or they are not.

Notation

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By tradition, a *set is denoted by a capital letter*, frequently one that will serve as a reminder of the contents of the set.

Roster notation (also called *listing notation*) is a method of describing a set by listing each element of the set inside the symbols { and }, which are called *set braces*.

In a listing of the elements of a set, *each distinct element is listed only once*, and the order of the elements doesn't matter.

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Example: N = {Mary, Chris, Joe, Jose, Linda}

Notation

The symbol $\in$ stands for "*is an element of* ", and

$\notin$ stands for "*is not an element of* ".

The **cardinal number** of a set *A* (or *cardinality*) is the number of elements in the set and is denoted by $n(A)$ or $|A|$.

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Example:

Thus, if *R* is the set of all letters in the name "Ramones," then *R* = {r, a, m, o, n, e, s} = {a, e, m, n, o, r, s}.

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Thus, if *R* is the set of all letters in the name "Ramones," then *R* = {r, a, m, o, n, e, s} = {a, e, m, n, o, r, s}.

Notice that m $\in$ *R*, x $\notin$ *R*, and *n(R)* = 7.

Notation

Two sets are **equal** if they contain exactly the same elements. *The order in which the elements are listed does not matter*.

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Two sets are **equal** if they contain exactly the same elements. *The order in which the elements are listed does not matter*.

Example:

If M is the set of all letters in the name "Moaners," then $M = \{m, o, a, n, e, r, s\}$.

Notation

Two sets are **equal** if they contain exactly the same elements. *The order in which the elements are listed does not matter*.

Example:

If M is the set of all letters in the name "Moaners," then M = {m, o, a, n, e, r, s}.

This set contains exactly the same elements as the set R of letters in the name "Ramones."

Therefore, M = R = {a, e, m, n, o, r, s}.

Notation

In the case of the set *G* of all negative real numbers, how can we list all the numbers?

Notation

In the case of the set *G* of all negative real numbers, no list, no matter how long, is capable of listing all members of the set; there is an infinite number of negative numbers.

In such cases, it is often necessary, or at least more convenient, to use **set-builder notation**, which lists ***the rules*** that determine whether an object is an element of the set rather than the actual elements.

Notation

A set-builder description of set G above is

$$G = \{x | x < 0 \quad \text{and} \quad x \in \Re\}$$

which is read as “the set of all *x* such that *x* is less than zero and *x* is a real number.”

Whatever is on the left side of the line is the general type of thing in the set, while the rules about set membership are listed on the right.

Example 2 – *Reading Set-Builder Notation*

Describe each of the following in words.

a. $\{x \mid x > 0 \text{ and } x \in \Re\}$

b. {persons | the person is a living former U.S. president}

c. {women | the woman is a former U.S. president}

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b. the set of all people such that the person is a living former U.S. president

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Solution:

a. the set of all *x* such that *x* is a positive real number

b. the set of all people such that the person is a living former U.S. president

c. the set of all women such that the woman is a former U.S. president

Notation

The set listed in part (c) of Example 2 has no elements; there are no women who are former U.S. presidents.

If we let W equal "the set of all women such that the woman is a former U.S. president," then $n(W) = 0$.

A set that has no elements is called an **empty set** and is denoted by Ø or by { }.

Since the empty set has no elements, $n(Ø) = 0$.

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A set that has no elements is called an **empty set** and is denoted by Ø or by { }.

Since the empty set has no elements, $n(\varnothing) = 0$.

In contrast, the set {0} is not empty; it has one element, the number zero, so $n(\{0\}) = 1$.

Universal Set and Subsets

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In some problems we announce the universal set beforehand.

Universal Set and Subsets

When *every element of one set is also a member of another set*, we say that the first set is a *subset* of the second;

Example: {p, i, n} is a subset of {p, i, n, e}.

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Alternatively, $A \subseteq B$ if A contains no elements that are not in B.

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In general, we say that *A* is a **subset** of *B*, denoted by $A \subseteq B$, if for every $x \in A$ it follows that $x \in B$.

Alternatively, $A \subseteq B$ if *A* contains no elements that are not in *B*.

If A contains an element that is not in B, then A is not a subset of B (symbolized as $A \nsubseteq B$).

Example 3 – *Determining Subsets*

Let *N* = {Adam, Christopher, Cindy, David, Elaine, Jane, Jonathan, Mark, Mary, Margaret, Sam,}. Determine whether the given set is a subset of *P*.

a. *A* = {Christopher, Cindy, David, Karen}

b. *B* = {Adam, Christopher, David, Jonathan, Mark, Sam}

c. *C* = { }

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a. *A* = {Christopher, Cindy, David, Karen}

b. *B* = {Adam, Christopher, David, Jonathan, Mark, Sam}

c. *C* = { }

Solution:

a. Karen $\notin$ N, hence A $\not\subseteq$ N

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b. *B* = {Adam, Christopher, David, Jonathan, Mark, Sam}

c. *C* = { }

Solution:

b. Since every element of *B* is also an element of *N*, *B* is a subset of *N*; $B \subseteq N$.

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Solution:

c. Does *C* contain an element that is not in *N*? No!
Therefore, *C* (an empty set) is a subset of *N; C* $\subseteq$ *N*.

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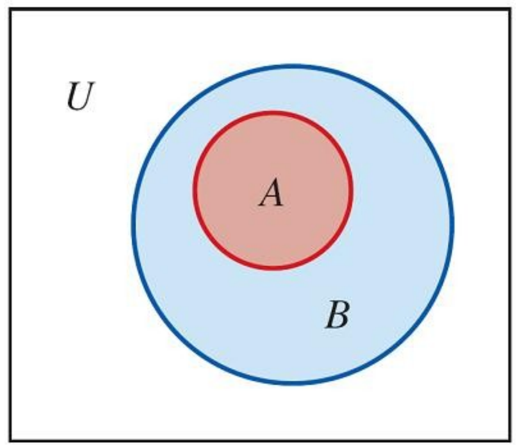
c. *C* = { }

Solution:

c. Does *C* contain an element that is not in *N*? No!
Therefore, *C* (an empty set) is a subset of *N*; $C \subseteq N$.
In general, the empty set is a subset of all sets; $\emptyset \subseteq S$ for any set *S*.

Universal Set and Subsets

We can express the relationship $A \subseteq B$ visually by drawing a Venn diagram, as shown in Figure 2.1.



A is a subset of *B*. $A \subseteq B$.

Figure 2.1

A **Venn diagram** consists of a rectangle, representing the universal set, and various closed figures within the rectangle, each representing a set.

Universal Set and Subsets

If two sets are equal, they contain exactly the same elements. It then follows that each is a subset of the other.

For example, if $A = B$, then every element of A is an element of B (and vice versa).

In this case, A is called an **improper subset** of B.

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For example, if $A = B$, then every element of A is an element of B (and vice versa).

In this case, A is called an **improper subset** of B. (Likewise, B is an improper subset of A.)

Every set is an improper subset of itself; for example, $A \subseteq A$.

Universal Set and Subsets

On the other hand, if *A* is a subset of *B* and *B* contains an element not in *A* (that is, $A \neq B$), then *A* is called a **proper subset** of *B*.

To indicate a proper subset, the symbol $\subset$ is used.

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To indicate a proper subset, the symbol $\subset$ is used.

While it is acceptable to write $\{1, 2\} \subseteq \{1, 2, 3\}$, the relationship of a proper subset is stressed when it is written $\{1, 2\} \subset \{1, 2, 3\}$.

Intersection of Sets

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If an element is in two sets *at the same time*, it is *in the intersection* of the sets.

Intersection of Sets

The **intersection** of set A and set B, denoted by $A \cap B$, is

$$A \cap B = \{x \mid x \in A \quad \text{and} \quad x \in B\}$$

The intersection of two sets consists of those elements that are common to both sets.

Intersection of Sets

Example: let

A = {Aquaman, Batman, Flash, Superman, Wonder Woman}

and

B = {Batman, Blue Beetle, Booster Gold, Fire, Flash},

their intersection is

$A \cap B =$

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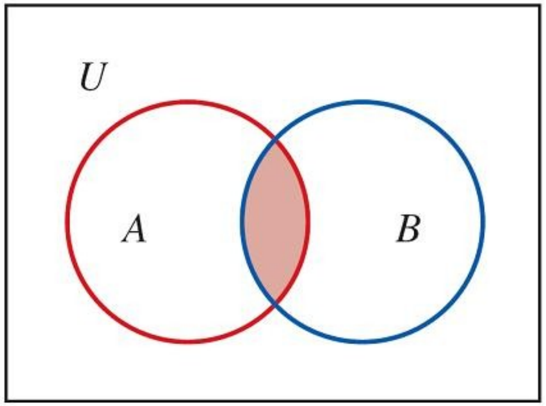
Venn diagrams are useful in depicting the relationship between sets. The Venn diagram in Figure 2.2 illustrates the intersection of two sets; the shaded region represents $A \cap B$.



The intersection $A \cap B$ is represented by the (overlapping) shaded region.

Figure 2.2

Mutually Exclusive Sets

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Consider an ordinary deck of playing cards:

Let D = {cards | the card is a diamond} and
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Certainly, *no* cards are both diamonds and spades *at the same time*; that is, $S \cap D = \emptyset$.

Mutually Exclusive Sets

Sometimes a pair of sets has no overlap.

Consider an ordinary deck of playing cards:

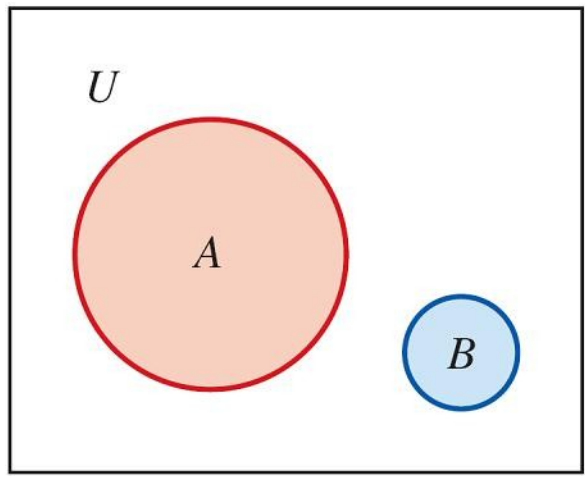
Let D = {cards | the card is a diamond} and
S = {cards | the card is a spade}.

Certainly, *no* cards are both diamonds and spades *at the same time*; that is, $S \cap D = \varnothing$.

Two sets A and B are **mutually exclusive** (or *disjoint*) if they have no elements in common, that is, if $A \cap B = \varnothing$.

Mutually Exclusive Sets

The Venn diagram in Figure 2.3 illustrates mutually exclusive sets.



Mutually exclusive sets have no elements in common ($A \cap B = \varnothing$).

Figure 2.3

Union of Sets

Union of Sets

The meaning of the word *or* is important to the concept of union. The **union** of two sets is a *new set* formed by joining those two sets together, just as the union of the states is the joining together of fifty states to form one nation.

Union of Sets

The **union** of set A and set B, denoted by $A \cup B$, is

$$A \cup B = \{x \mid x \in A \quad \text{or} \quad x \in B\}$$

The union of A and B consists of all elements that are in either A or B or both, that is, all elements that are in at least one of the sets.

Union of Sets

Example: given the sets

A = {Daryl, Glenn, Rick} and
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Union of Sets

Example: given the sets

A = {Daryl, Glenn, Rick} and
B = {Maggie, Michonne},

their union is

$A \cup B$ = {Daryl, Glenn, Maggie, Michonne, Rick},

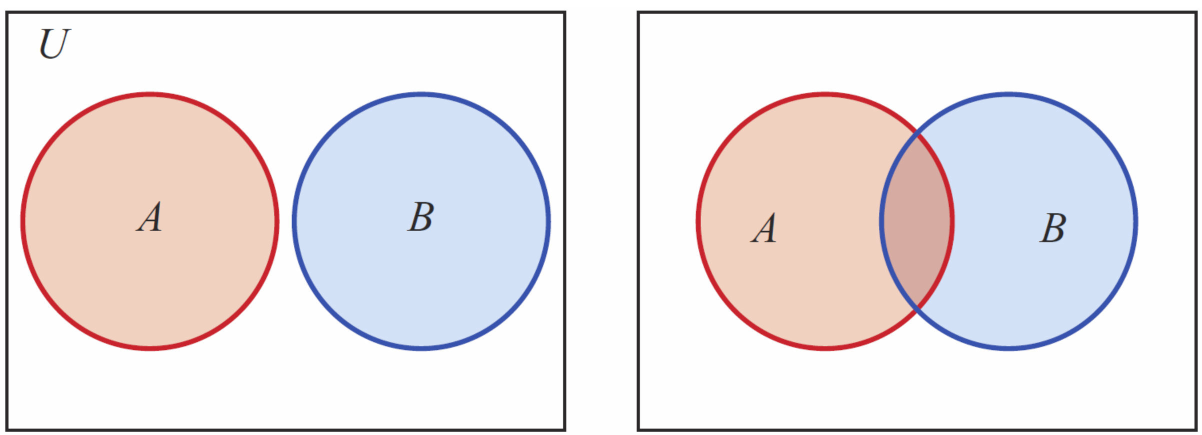
and their intersection is

$A \cap B = \varnothing$.

Note that because they have no elements in common, A and B are *mutually exclusive sets*.

Union of Sets

The Venn diagram in Figure 2.4 illustrates the union of two sets; part (a) depicts *mutually exclusive sets*, whereas part (b) depicts *nonmutually exclusive* (*overlapping*) *sets*; the entire shaded region represents $A \cup B$.



(a) mutually exclusive sets (b) nonmutually exclusive sets

The union $A \cup B$ *is* represented by the (entire) shaded region.

Figure 2.4

Example 4 – *Finding the Intersection and Union of Sets*

Given the sets $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$, find the following.

a. $A \cap B$ (the intersection of A and B)

b. $A \cup B$ (the union of A and B)

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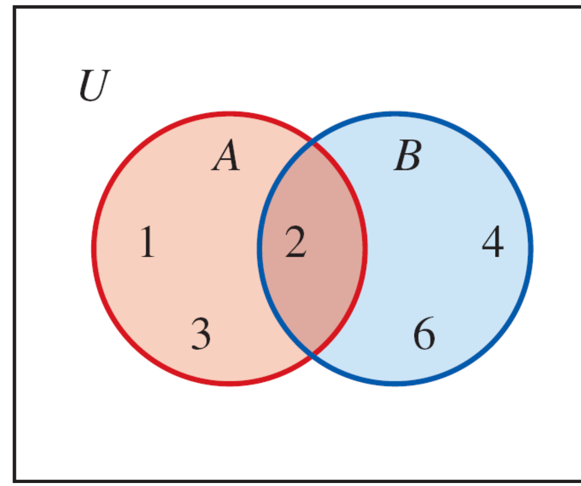

Solution:

a. The intersection of two sets consists of those elements that are common to both sets; therefore, we have

$$A \cap B = \{1, 2, 3\} \cap \{2, 4, 6\} = \{2\}$$

Example 4 – *Finding the Intersection and Union of Sets*

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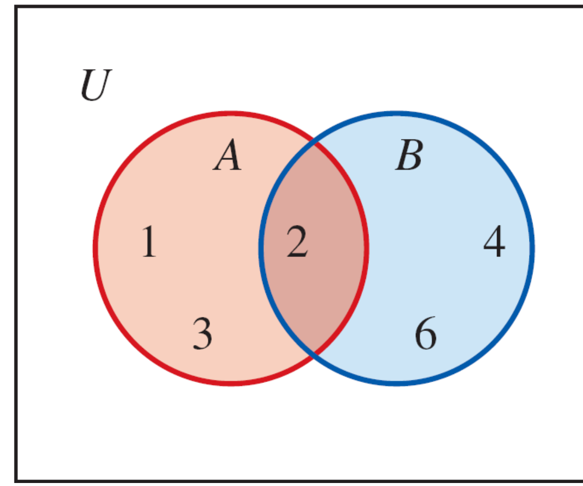

Solution:

b. The union of two sets consists of all elements that are in at least one of the sets; therefore, we have

$$A \cup B = \{1, 2, 3\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\}$$

Union of Sets

Cardinal Number Rule for the Union/Intersection of Sets

For any two sets A and B, the number of elements in their union is $n(A \cup B)$, where

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where $n(A \cap B)$ is the number of elements in their intersection.

As long as any three of the four quantities in the general formula are known, the missing quantity can be found by algebraic manipulation.

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where $n(A \cap B)$ is the number of elements in their intersection.

if A and B have elements in common, these elements will be counted twice (once as a part of A and once as a part of B).

Example 5 – *Analyzing the Composition of a Universal Set*

Given $n(U) = 169$, $n(A) = 81$, and $n(B) = 66$, find the following:

a. If $n(A \cap B) = 47$, find $n(A \cup B)$ and draw a Venn diagram depicting the composition of the universal set.

b. If $n(A \cup B) = 147$, find $n(A \cap B)$ and draw a Venn diagram depicting the composition of the universal set.

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Solution:

a. Substituting the three given quantities, we have:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 81 + 66 - 47$$

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Solution:

a. Substituting the three given quantities, we have:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 81 + 66 - 47 = 100$$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

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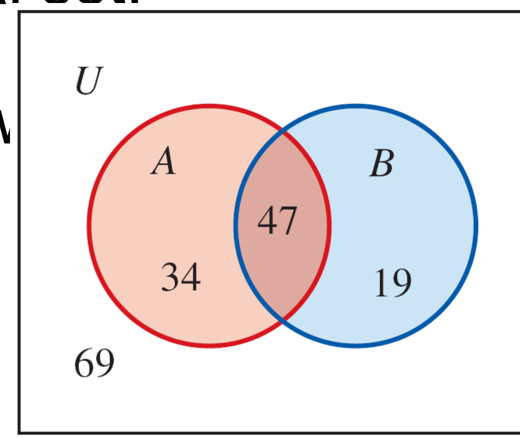

Solution:

a. Substituting the three given quantities, we have:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 81 + 66 - 47 = 100$$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

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b. If $n(A \cup B) = 147$, find $n(A \cap B)$ and draw a Venn diagram depicting the composition of the universal set.

Solution:

b. Substituting the three given quantities, we have:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$147 = 81 + 66 - n(A \cap B)$$

Example 5 – *Analyzing the Composition of a Universal Set*

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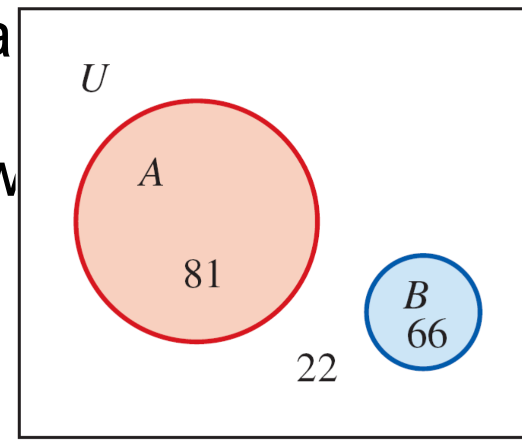

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Complement of a Set

Complement of a Set

In certain situations, it might be important to know how many things are *not* in a given set.

For instance, when playing cards, you might want to know how many cards are not ranked lower than a five; or when taking a survey, you might want to know how many people did not vote for a specific proposition.

Complement of a Set

The set of all elements in the universal set that are *not* in a specific set is called the *complement* of the set.

Complement of a Set

The **complement** of set A, denoted by A' (read "A prime" or "the complement of A"), is

$$A' = \{x \mid x \in U \quad \text{and} \quad x \notin A\}$$

The complement of a set consists of all elements that are in the universal set, but not in the given set.

Complement of a Set

Example: given that

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and

$A = \{1, 3, 5, 7, 9\}$, the complement of A is $A' =$

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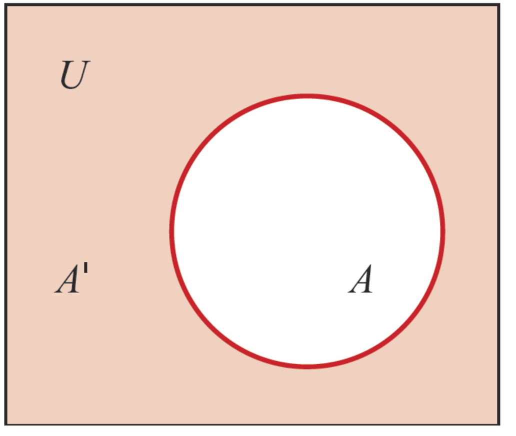
What is the complement of A'?



The complement A' is represented by the shaded region. **Figure 2.8**

Complement of a Set

Example: given that

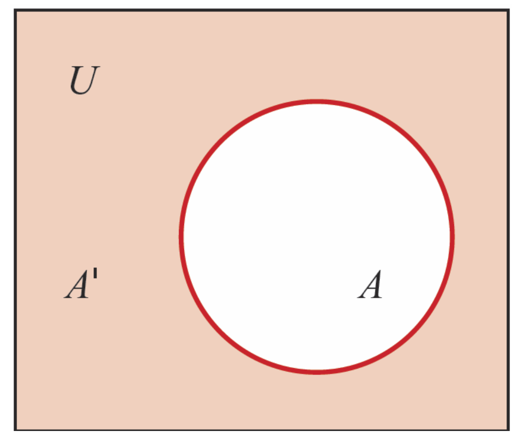
U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and

A = {1, 3, 5, 7, 9}, the complement of A is A' = {2, 4, 6, 8}.

What is the complement of A'?

Just as $-(-x) = x$ in algebra,

$(A')' = A$ in set theory.



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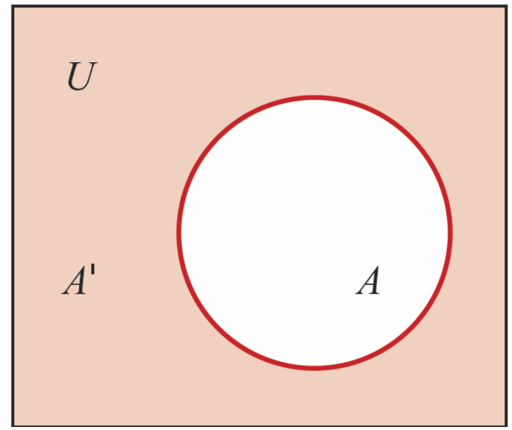
Complement of a Set

Suppose *A* is a set of elements, drawn from a universal set *U*.

If $x \in U$, then <u>exactly one of the following</u> must be true:

(1) $x \in A$ or

(2) $x \notin A$



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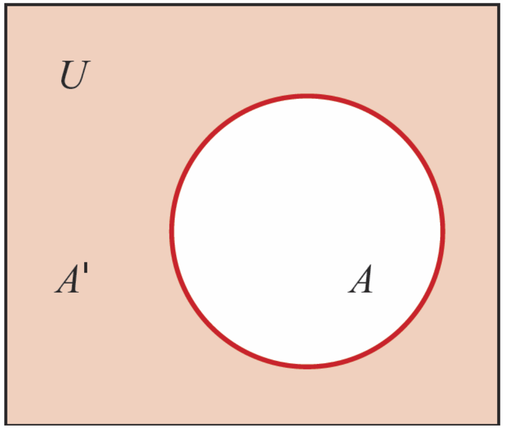

If $x \in U$, then <u>exactly one of the following</u> must be true:

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So no element of the universal set can be in both *A* and *A′* at the same time.

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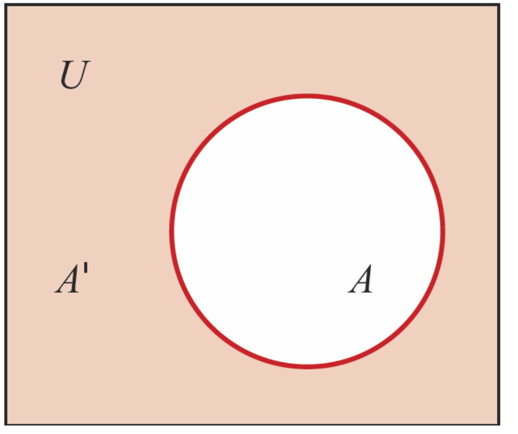

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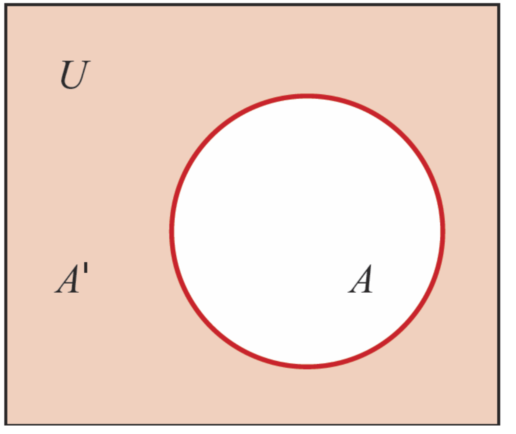

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So no element of the universal set can be in both A and A' at the same time.

Therefore, A and A' are mutually exclusive sets whose union equals the entire universal set.

In addition, $n(U) = n(A) + n(A')$

Complement of a Set

To find the cardinal number of a set, we can subtract the cardinal number of its complement from the cardinal number of the universal set; that is, $n(A) = n(U) - n(A')$.

Cardinal Number Rule for the Complement of a Set

For any set A and its complement A',

$$n(A) + n(A') = n(U)$$

where U is the universal set.

Alternatively,

$$n(A) = n(U) - n(A') \quad \text{and} \quad n(A') = n(U) - n(A)$$

Example 7 – *Using the Complement Rule*

How many letters in the alphabet precede the letter w?

Example 7 – *Using the Complement Rule*

How many letters in the alphabet precede the letter w?

Solution:

Rather than counting all the letters that precede w, we will take a shortcut by counting all the letters that do *not* precede w.

Let L = {letters | the letter precedes w}, hence

L' = {letter | the letter does not precede w}.

Now L' = {w, x, y, z}, and $n(L') = 4$; therefore, we have

$$n(L) = n(U) - n(L')$$

Complement Formula

$$= 26 - 4$$

$$= 22$$

There are twenty-two letters preceding the letter w.

Shading Venn Diagrams

Shading Venn Diagrams

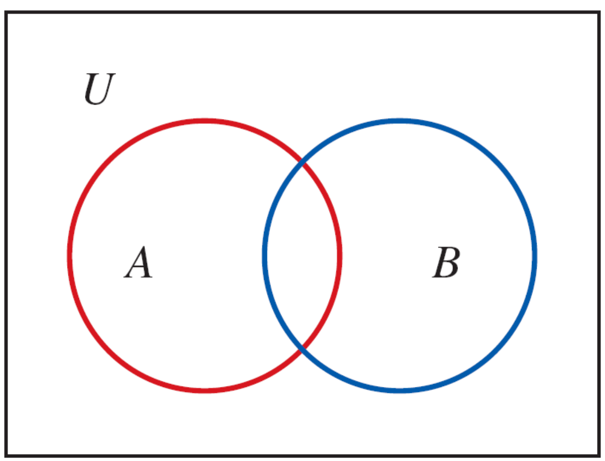
In an effort to visualize the results of operations on sets, it may be necessary to shade specific regions of a Venn diagram.

The next example shows a systematic method for shading the intersection or union of any two sets.

Example 8 – *Shading Venn Diagrams*

On a Venn diagram, shade in the region corresponding to the indicated set.

a. $A \cap B'$



Two overlapping circles.

Figure 2.9

b. $A \cup B'$

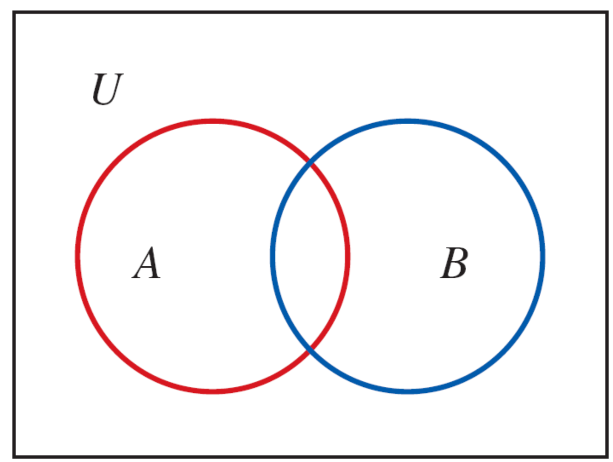


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Figure 2.9

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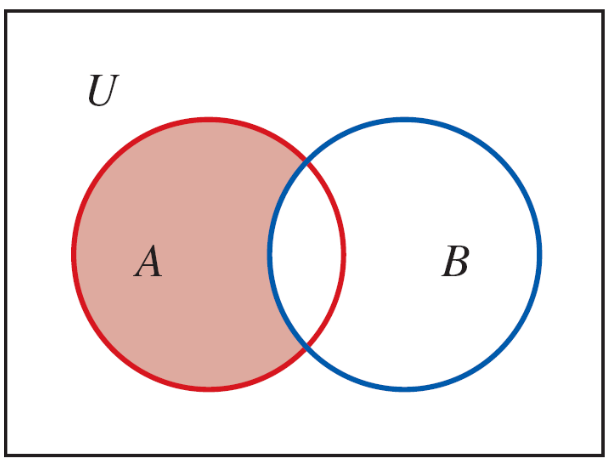

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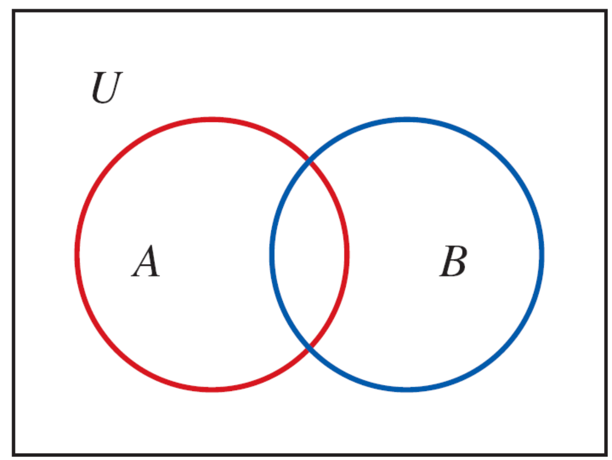

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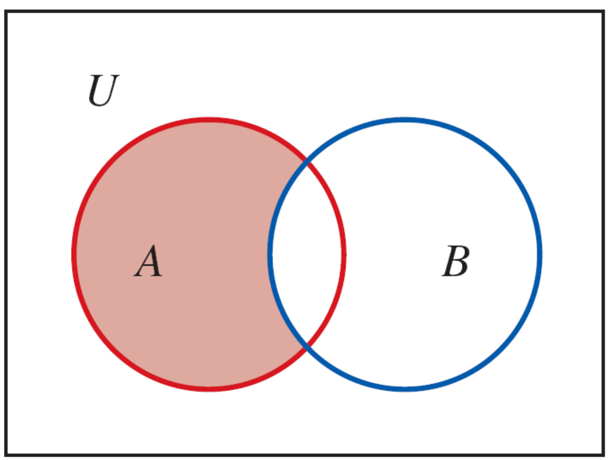

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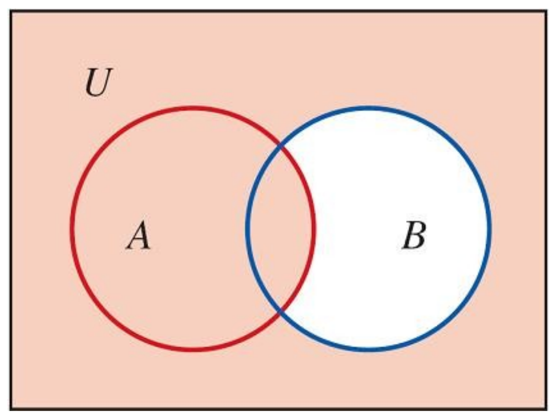


Example 8 – *Solution*

cont'd

a. $A \cap B'$: The two "components" of the operation $A \cap B'$ are "A" and "B'."

Shade each of these components in contrasting ways:

- shade one of them, say A, with horizontal lines, and

- the other with vertical lines as in Figure 2.10.

Be sure to include a legend, or key, identifying each type of shading.

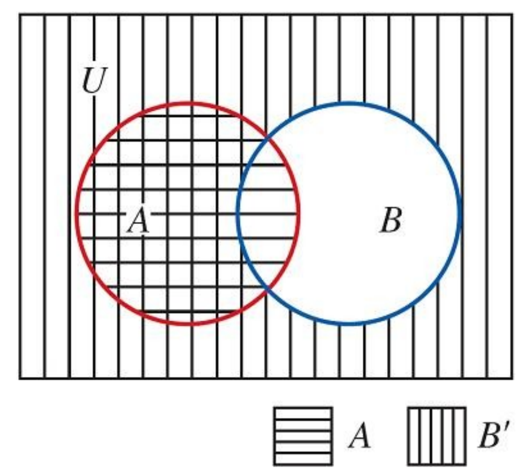


Figure 2.10

Example 8 – *Solution*

cont'd

To be in the intersection of two sets, an element must be in *both* sets at the same time.

Therefore, the intersection of A and B' is the region that is shaded in *both* directions (horizontal and vertical) at the same time.

A final diagram depicting $A \cap B'$ is shown in Figure 2.11:

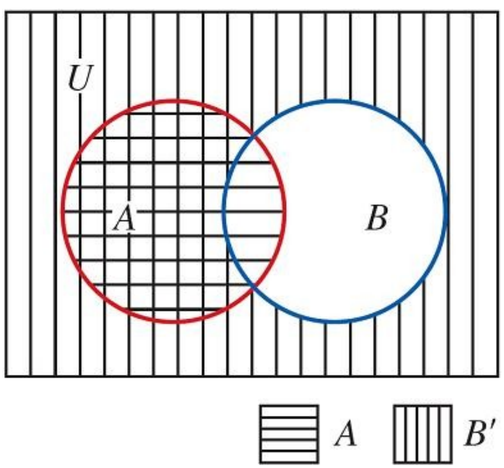
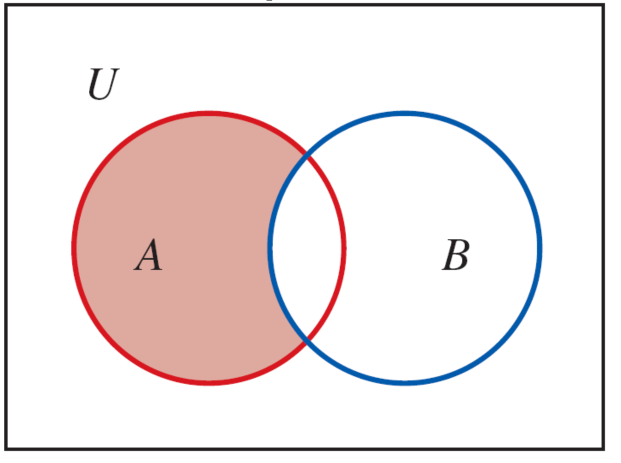


Figure 2.10



$A \cap B'$

Figure 2.11

Example 8 – *Solution*

cont'd

b. Refer to Figure 2.10. To be in the union of two sets, an element must be in *at least one* of the sets. Therefore, the union of A and B' consists of all regions that are shaded in *any* direction whatsoever (horizontal or vertical or both). A final diagram depicting $A \cup B'$ is shown in Figure 2.12.

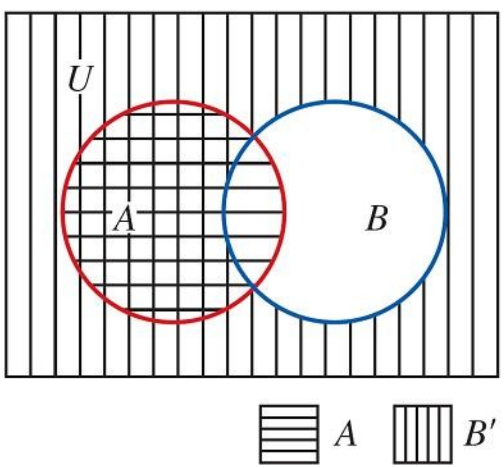
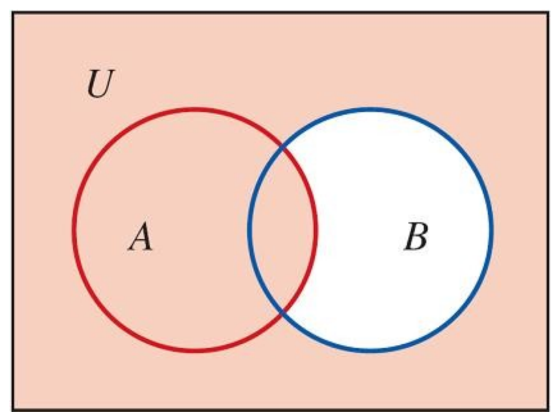


Figure 2.10



$A \cup B'$

Figure 2.12