MTH 21 and MTH 21.5 Midterm Exam Review Problems

Chapter 2

1. Let S = {1, 2, 3, a, b, c}

Determine which of the following sets are *subsets* of S (select all that apply):
(a) {1, a, c}
(b) {}
(c) {1, 2, 3, 4}
(d) {a, b, c, d}
(e) {2, 3, a}

2. Find the *cardinality* (*cardinal number*) of the set B = { 1, 2, 3, {a, b, c}, 6}

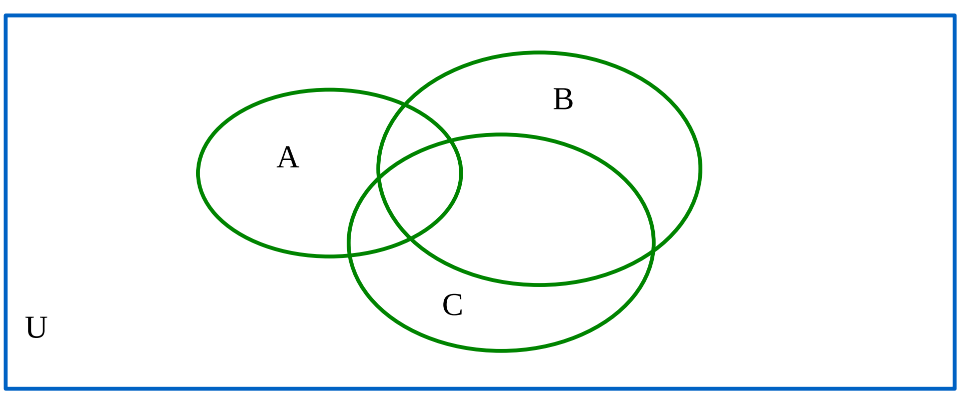
3. Let A = {1, 2, 3, 4, 5, 6} and B = {1, a, 2, b, 3, c}. Select all the statements that are true:

(a) $A \subset B$
(b) $\{1, 2, 3\} \subset B$
(c) $A \subseteq A$
(d) $5 \notin A$
(e) $56 \in A$
(f) $\varnothing \in A$
(g) $\varnothing \subseteq A$
(h) $\{1, 2, 3\} \in B$
(i) $b \in B$
(j) n(A) = n(B) or |A| = |B|
(k) $B \subset A$

4. Let universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, set A = {2, 5, 8, 9, 10}, and set B = {1, 3, 5, 7, 9} Find

(a) $A \cup B$ (b) $A \cap B$ (c) A' (d) $A \cap (B \cup A')$

5. Given the Venn diagram of the universal set U, sets A, B and C, shade the area corresponding to $A \cap (B \cup C')$



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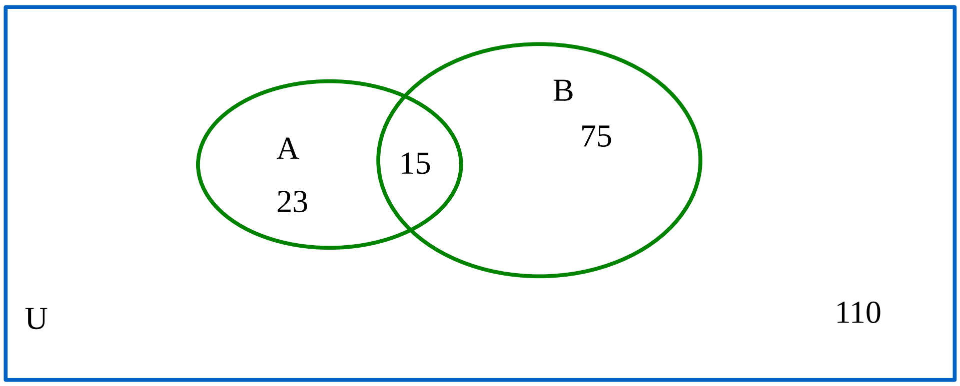
6. The number of elements in each region are shown in the Venn diagram below.



Find

(a) $n(A \cap B)$ (b) $n(A \cup B)$ (c) $n((A \cup B)')$ (d) $n(U)$

7. We know the following about the sets A and B: $n(A) = 13$, $n(B) = 21$, and $n(A \cap B) = 3$.
Find $n(A \cup B)$.

8. We know the following about the sets A and B: $n(A \cup B) = 56$, $n(B) = 25$, and $n(A \cap B) = 13$.
Find $n(A)$.

9. The security password to a security deposit box should consist of 5 symbols.
The first symbol is a digit (0-9), the next two symbols are letters of the alphabet (lower case, repetitions are allowed), and the last two symbols are selected from these special symbols: {*, &, !, ~, (,), %, $, #, _, -, @}. Repetitions of special symbols are not allowed.
Here are the examples of valid passwords: 7fd%@ 6jj$(0kt_&
Here are the examples of invalid passwords: 6gh** (two asterisks at the end is the error)
8_jh& (the special symbol came instead of the letter of the alphabet)
How many all the possible and valid security passwords are there in total?

10. Two dice are rolled. How many possible outcomes are there if the order of numbers coming up on the dice doesn't matter to us, i.e. 2 on the first and 3 on the second is the same as 3 on the first and 2 on the second?

11. An ice cream parlor serves 3 toppings for each ice cream for free. There are 8 available toppings. In how many ways the toppings can be selected if the same topping cannot be chosen more than once, but the order in which the toppings are applied doesn't matter to us.

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12. In how many ways four letter words can be made from the 10 first letters of English alphabet? (words do not necessary need to be meaningful, however the repetition of letters is not allowed)

13. Evaluate P(8,5)

14. Evaluate C(8,5)

15. Consider sets S = {Jane, Maria, Kimberley, Joanna, Helena, Tracy} and T = {Smith, Johnson, Leon, Simon, Nunez}. Do these sets have one-to-one correspondence? Explain.

16. Samantha told me that sets A and B have the same cardinal number (same cardinality). Can I conclude that these two sets are equivalent?

17. Is the set of all students in our class countable?

One extra: For a test with 10 questions, the questions are selected from the pool of 18 questions. In how many different ways can the questions in this test be arranged?
(the questions are numbered and the order of questions is important)

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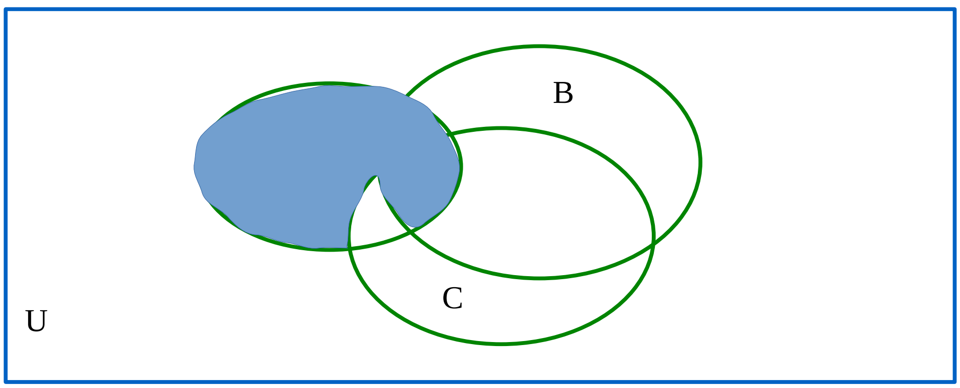
Answers:

Chapter 2
1. (a), (b), and (e)
2. n(B) = |B| = 4
3. (b), (c), (g), (i), (j)
4. (a) $A \cup B = \{1, 2, 3, 5, 7, 8, 9, 10\}$
(b) $A \cap B = \{5, 9\}$
(c) $A' = \{1, 3, 4, 6, 7\}$
(d) $A \cap (B \cup A') = \{5, 9\}$

5.



6. (a) $n(A \cap B) = 15$, (b) $n(A \cup B) = 23 + 15 + 75 = 113$,
(c) $n((A \cup B)') = 110$, (d) $n(U) = 110 + 23 + 15 + 75 = 223$

7. Using the formula $n(A \cup B) = n(A) + n(B) – n(A \cap B)$, we can find $n(A \cup B) = 13 + 21 – 3 = 31$
Answer: $n(A \cup B) = 31$

8. Using the formula $n(A \cup B) = n(A) + n(B) – n(A \cap B)$, we get: $56 = n(A) + 25 – 13$,
hence $n(A) = 44$
Answer: $n(A) = 44$

9. $10 \times 26 \times 26 \times 12 \times 11 = 892,320$

10. $6 \times 6 = 36$ is the number of all possible combinations of numbers on two dice, and since the order is not important to us, <u>half of those outcomes are duplicates</u>, i.e. 2 on the first and 3 on the second, 3 on the first and 2 on the second should be counted as one outcome, not as two outcomes.
Final answer: 18 outcomes

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11. This is a 3-combination out of 8, i.e. C(8,3) = $\frac{8!}{3!(8-3)!}=\frac{6\times7\times8}{2\times3}=56$ ways

12. Since order is important to us, and no repetitions are allowed, it is a 4-permutation of 10 letters, i.e. P(10,4) = $\frac{10!}{(10-4)!}=7\times8\times9\times10=5{,}040$ ways

Another way to solve this problem:
we have 4 decisions to make: the each of these places we need to select a letter from a pool of 10 letters

_____ _____ _____ _____

We can select one letter out of 10 for the first place.
Then we can select one letter our of 9 remaining letters for the second place (remember: no repetitions, so once a letter is used, it is no longer available!)
And so forth. So we will be getting:

__10__ __9__ __8__ __7__
Taking their product: 10 * 9 * 8 * 7 = 5,040

13. $P(8,5)=\frac{8!}{(8-5)!}=4\times5\times6\times7\times8=6{,}720$

14. $C(8,5)=\frac{8!}{5!(8-5)!}=\frac{6\times7\times8}{2\times3}=84$

15. There is no one-to-one correspondence between sets S and T because they have different cardinalities, i.e. n(S) ≠ n(T)

16. Yes, I can. Recall this "theorem" from our lecture slides (Section 2.5):
Given two sets *A* and *B*, if any one of the following statements is true, then the other statements are also true:
1. There exists a one-to-one correspondence between the elements of *A* and *B.*
2. *A* and *B* are equivalent sets.
3. *A* and *B* have the same cardinal number; that is, *n*(*A*) = *n*(*B*).

17. Yes, it is countable, because the set of all students in our class is a finite set!

One extra:
Either draw 10 boxes and put numbers 18 through 9 there, then multiply them (applying the Fundamental principle of counting)
or
say that we have 10-permutation from 18 questions, hence it is ${}_{18}P_{10}$ = P(18,10) =
$\frac{18!}{(18-10)!}=158{,}789{,}030{,}400$ different ways the questions in this test can be arranged