MTH 21 and MTH 21.5 Midterm Exam Review Problems

Chapter 5

1. Review the terminology and all the formulas met in this chapter.

2. (Simple interest formula) Anna and Jack buy a three-year CD that pays 4.7% simple interest from the bank AAA for $70,000. Find the interest that the investment earns and the value of the CS at the end of this term.

3. (Simple interest formula) Company B asked bank C for a short-term loan of $27,000 at 8.2% from December 12th to Februray 10th. Find the amount they need to pay back to bank C at the end of the loan (don't forget to include the interest!).

4. (Simple interest formula) Find the amount of money that must be invested now at a 4.9% simple interest so that it will be worth $2,300 in one and a half year.

5. (Simple interest formula) John bought a $750 TV with a help of an add-on interest loan with 6% simple interest. He maid $50 down payment, and the term of the loan is 15 months. What is the monthly payment?

6. (Simple interest formula) Ed has made an investment that will return a fixed percentage simple interest. He knows that he started with 8000 dollars on March 10, and that on May 9, the investment is worth $8,120.98 dollars. What rate of interest is he receiving?

7. (Compound interest formula) Frank borrows $11,000 at 9.6% annual interest, compounded once per year, when he begins college. Four years later, how much will he owe?
(Assume that he makes no payments during the four years.)

8. (Compound interest formula) Frank borrows $11,000 at 9.6% annual interest, <u>compounded monthly</u>, when he begins college. Four years later, how much will he owe?
(Assume that he makes no payments during the four years.)

9. (Compound interest formula) Gina wants to buy a house in 10 years and would like to have $120,000 for her down payment by then. How much should she invest now, at 7.2% interest compounded quarterly, to have the amount needed for the down payment in 10 years?

10. (Compound interest formula) Miguel deposited $3,950 in an account, and 4 years later the account balance is $8,800.
If interest is compounded monthly, what is the rate of interest per month

11. (Compound interest formula) In general, which rate yields the larger amount in one year?

(A) 7% compounded semi-annually
(B) 6.97% compounded quarterly

MTH 21 and MTH 21.5 Midterm Exam Review Problems

12. (Annuity) You decided to open an ordinary annuity account with an annual interest rate of 4.36% compounded monthly, and will be making monthly payments of $325.00 into the account.
How much will you have in the account after 4 years?
How much interest will you earn?

13. (Annuity) You decided to open an ordinary annuity account with an annual interest rate of 3.55% compounded quarterly, and will be making monthly deposits of $1,590.00 into the account.
How many years to you need to continue to make the deposits so that you will have at least $107,000.00?

14. (Annuity) Jack wants to open an ordinary annuity account at bank A, that offers 5.35% annual interest rate, for 15 years. What monthly deposits should he plan to make if he wants to have $25,800.00 by the end of the term?

15. (Amortized loans) Samuel and Joanna decided to purchase a $350,000 house using a down payment of $18,000.00. The bank offered them 5% interest rate and 25 years loan. Find:

1) their monthly payment
2) the total interest they will payments
3) the approximate balance after 7 years
4) the approximate balance after 22 years

16. (Amortized loans) A photo and video store sells a photo camera for $400 down and monthly payments of $80 for the next 3 years. If the interest rate is 2.25% annually, find the cost of the photo camera.

MTH 21 and MTH 21.5 Midterm Exam Review Problems

Answers:

1. self-study

2. t = 3 years, r = 0.047, and P = \$70,000

The interest I = \$70,000 × 0.047 × 3 = \$9,870

The value of the CD at the end of the term: \$70,000 + \$9,870 = **\$79,870.**

3. P = \$27,000, r = 0.082, t =

Count the days! "to" doesn't include the last day. Company B will need to return the money with the fee on February 10th.

(31-11) + 31 + 9 = 60 days

$$FV = P(1+rt) = 27{,}000\left(1+0.082\times\frac{60}{360}\right) = 27{,}369 \quad (\$)$$

Company B will have to pay \$27,369 on February 10th to bank C.

4. r = 0.049, t = 1.5, FV = \$2,300. Substitute into the formula $FV = P(1+rt)$ to find the P.

P = 2142.524453…

In order to make sure that the FV is at least \$2,300, **\$2,142.53 must be invested**!

5. Loan = 750 – 50 = 700, time t = $\frac{15}{12} = 1.25$ years, r = 0.06. Find the FV:

$$FV = P(1+rt) = 700(1+0.06\times 1.25) = 752.50 \quad (\$)$$

Monthly payment = $752.50 \div 15 = 50.16666\ldots$, rounding off two cents: ≈ 50.17 (\$)

John will pay \$50.17 monthly (the last payment will be slightly different).

6. P = \$8,000, FV = \$8,120.98, time t = … count the days: (31-9) + 30 + 9 = 61 days.

Hence, time t = $\frac{61}{360}$. Use formula $FV = P(1+rt)$ to find r:

$r = 0.0892475\ldots \approx 0.089$ or **8.9%**

7. i = 0.096 (since the interest is compounded annually), t = 4 years = n

Using formula $FV = P(1+i)^n = 11{,}000(1+0.096)^4 = 15{,}872.11867\ldots \approx 15{,}872.12$ (\$)

At the end of 4 years, Frank owes \$15,872.12

8. i = $\frac{0.096}{12} = 0.08$ (since the interest is compounded monthly), t = 4 years, hence n = 4 × 12 = 48

Using formula $FV = P(1+i)^n = 11{,}000(1+0.08)^{48} = 442{,}316.3045657\ldots \approx 442{,}316.30$ (\$)

At the end of 4 years, Frank owes \$442,316.30

9. Periodic rate (quarterly) is i = $\frac{r}{4} = \frac{0.072}{4} = 0.018$, number of periods n = 10 × 4 = 40, and FV = \$120,000.

Using formula $FV = P(1+i)^n$: $120{,}000 = P(1+0.018)^{40}$ find P = $\frac{120{,}000}{(1+0.018)^{40}} \approx 58{,}785.49$ (\$)

Gina needs to invest \$58,785.49 (she will get a little over \$120,000 in 10 years)

MTH 21 and MTH 21.5 Midterm Exam Review Problems

10. P = \$3,950, FV = \$8,800, n = 4 × 12 = 48 months. Using formula $FV = P(1+i)^n$:

$8{,}800 = 3{,}950(1+i)^{48}$, find i: $i = \sqrt[48]{\frac{8{,}800}{3{,}950}} - 1 \approx 0.01683$ or 1.683%

Answer: the monthly interest rate is approximately 1.683%

11. We will find the annual yield for each of the rates. We will do this by using the following equation for each of the items: $FV(compound\,interest) = P(1+i)^n = P(1+rt) = FV(simple\,interest)$

(A) i = $\frac{0.07}{2} = 0.035$, n = 1 × 2, t = 1 year. Hence we get $P(1+0.035)^2 = P(1+r\times 1)$,

cancel P from both sides: $(1+0.035)^2 = 1+r$, then r = 0.071225 or **7.1225%**

(B) i = $\frac{0.0697}{4} = 0.017425$, n = 1 × 4 = 4, t = 1 year. Hence we get $P(1+0.017425)^4 = P(1+r\times 1)$.

cancel P from both sides: $(1+0.017425)^4 = 1+r$, then r ≈ 0.071543 or **7.1543%**

The annual yield in (B) is higher.

12. Use the formula $FV = pymt\frac{[(1+i)^n - 1]}{i}$, with *pymt* = 325, n = 4 × 12 = 48 and i = $\frac{0.0436}{12}$

Hence, $FV = \frac{325\times\left[\left(1+\frac{0.0436}{12}\right)^{48} - 1\right]}{\frac{0.0436}{12}} = 325\times\left[\left(1+\frac{0.0436}{12}\right)^{48} - 1\right]\times\left(\frac{12}{0.0436}\right) \approx 17{,}009.32$ (\$)

You will have \$17,009.3 in 4 years.

You will earn \$17,009.32 - \$325 × 48 = **\$1,409.32**

13. *pymt* = \$1,590, i = $\frac{0.0355}{4} = 0.008875$, FV = \$107,000.00. Using formula

$FV = pymt\frac{[(1+i)^n - 1]}{i}$:

$120000 = 1{,}590\frac{[(1+0.008875)^n - 1]}{0.008875}$,

multiply both sides by $\frac{0.008875}{1{,}590}$: $\frac{120000\times 0.008875}{590} = (1+0.008875)^n - 1$

add 1 to both sides: $\frac{120000\times 0.008875}{1{,}590} + 1 = (1+0.008875)^n$

evaluate the left side, add 1 and 0.008875 on the right side: $1.669811321 = (1.008875)^n$

guess n:

if n = 2, then the right side is $(1.008875)^2 = 1.017828766$ - too far from 1.669811321

if n = 20, then the right side is $(1.008875)^{20} = 1.193293311$ - closer to 1.669811321

try n = 50: $(1.008875)^{50} = 1.555492838$ - even more close to 1.669811321

try n = 60: $(1.008875)^{60} = 1.69918873$ - a little over 1.669811321

try n = 56: $(1.008875)^{56} = 1.640182515$

ok, then try n = 59: $(1.008875)^{59} = 1.684241091$ a little over 1.669811321, but so far the closest to it

try n =58: $(1.008875)^{58} = 1.669424944$ - this will be the one we stop, as it is the closest we can get to 1.669811321
58 quarters stands for 14.5 years (58 ÷ 4 = 14.5).

Answer: 15 years (if we count whole years).

14. Periodic rate i = $\frac{0.0535}{12}$, the number of periods n = 15 × 12 = 180, and FV = $25,800.00.

Using formula $FV = pymt\frac{[(1+i)^n - 1]}{i}$: $25{,}800 = pymt\frac{\left[\left(1+\frac{0.0535}{12}\right)^{180} - 1\right]}{\left(\frac{0.0535}{12}\right)}$

solving for *pymt*:

multiply both sides by denominator: $25{,}800 \times \left(\frac{0.0535}{12}\right) = pymt\left[\left(1+\frac{0.0535}{12}\right)^{180} - 1\right]$

divide both sides by expression inside [ ] : $\frac{25{,}800 \times \left(\frac{0.0535}{12}\right)}{\left[\left(1+\frac{0.0535}{12}\right)^{180} - 1\right]} = pymt$.

Then use calculator: *pymt* ≈ $93.73. To guarantee that he will have at leas $25,800.00 by the end of 15 years we can add one more cent to $93.73!

Answer: Jack will need to make $93.74 monthly deposits into the account.

15. in 1) we will use the formula $FV(annuity) = pymt\frac{((1+i)^n - 1)}{i} = P(1+i)^n = FV(compound\ interest)$ to find the monthly payments.

= $\frac{0.05}{12}$, n = 25 × 12 = 300, P = $350,000.00 - $18,000.00 = $332,000.00

$$pymt\frac{\left(\left(1+\frac{0.05}{12}\right)^{300} - 1\right)}{\frac{0.05}{12}} = 332{,}000.00\left(1+\frac{0.05}{12}\right)^{300}$$

multiply both sides by $\frac{0.05}{12}$: $pymt\left(\left(1+\frac{0.05}{12}\right)^{300} - 1\right) = 332{,}000.00\left(1+\frac{0.05}{12}\right)^{300} \times \frac{0.05}{12}$

then divide both sides by $\left(\left(1+\frac{0.05}{12}\right)^{300} - 1\right)$: $pymt = \frac{332{,}000.00\left(1+\frac{0.05}{12}\right)^{300} \times \frac{0.05}{12}}{\left(\left(1+\frac{0.05}{12}\right)^{300} - 1\right)} \approx 1{,}940.84$ ($)

Their monthly payment is $1,940.84

MTH 21 and MTH 21.5 Midterm Exam Review Problems

2) they will pay \$1940.84 × 300 = \$585,252.00
the loan is \$332,000.00.
Hence **they will pay** \$585,252.00 - \$332,000.00 = **\$250,252.00 interest**

3) for both 3) and 4) we will use the Unpaid Balance Formula:

unpaid balance = current value of loan amount – current value of annuity ≈ $P(1+i)^n - pymt\frac{(1+i)^n-1}{i}$

7 years = 7 × 12 = 84 periods , hence

their **unpaid balance** ≈ $332000\times\left(1+\frac{0.05}{12}\right)^{84}-\frac{1940.84\times\left(\left(1+\frac{0.05}{12}\right)^{84}-1\right)}{\frac{0.05}{12}}\approx 276{,}066.11$ **(\$)**

4) 22 years = 22 × 12 = 264 periods

their **unpaid balance** ≈ $332000\times\left(1+\frac{0.05}{12}\right)^{264}-\frac{1940.84\times\left(\left(1+\frac{0.05}{12}\right)^{264}-1\right)}{\frac{0.05}{12}}\approx 64{,}756.94$ **(\$)**

16. I = $\frac{0.0225}{12}=0.001875$, n = 3 × 12 = 36 periods,

use formula $FV(annuity)=pymt\frac{((1+i)^n-1)}{i}=P(1+i)^n=FV(compound\,interest)$ to find P, the loan.

$$80\frac{((1+0.001875)^{36}-1)}{0.001875}=P(1+0.001875)^{36}\quad,\quad P=\frac{80\frac{((1+0.001875)^{36}-1)}{0.001875}}{(1+0.001875)^{36}}\approx 2{,}782.43\ (\$)$$

Hence **the price of the photo camera was** \$2,782.43 + \$400 = **\$3,182.43**