Chapter 20
Additional Topics in Trigonometry

- Section 20.6
  The Inverse Trigonometric Functions

The Inverse Trigonometric Functions

By solving the equation

$y = \sin x$ for the independent variable x, we will get:

$x = \sin^{-1} y, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

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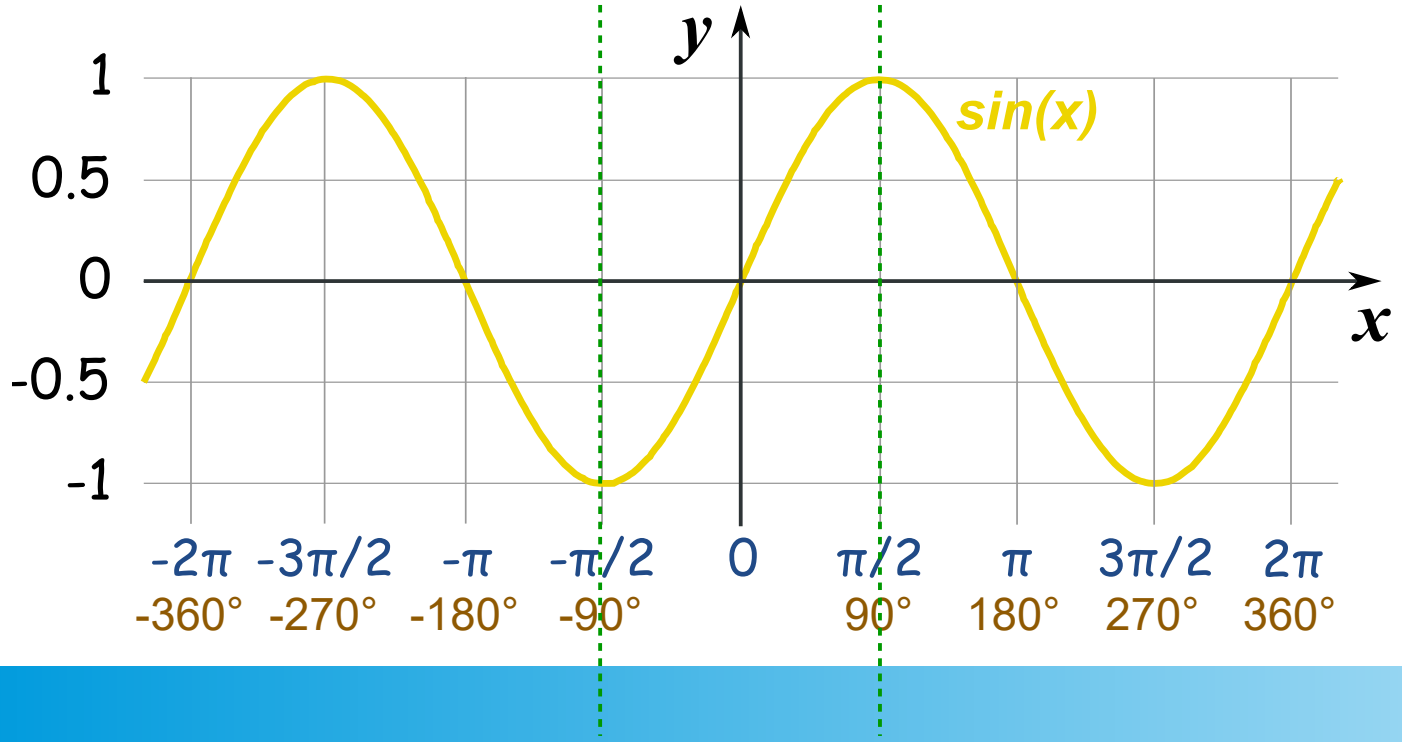

The Inverse Trigonometric Functions

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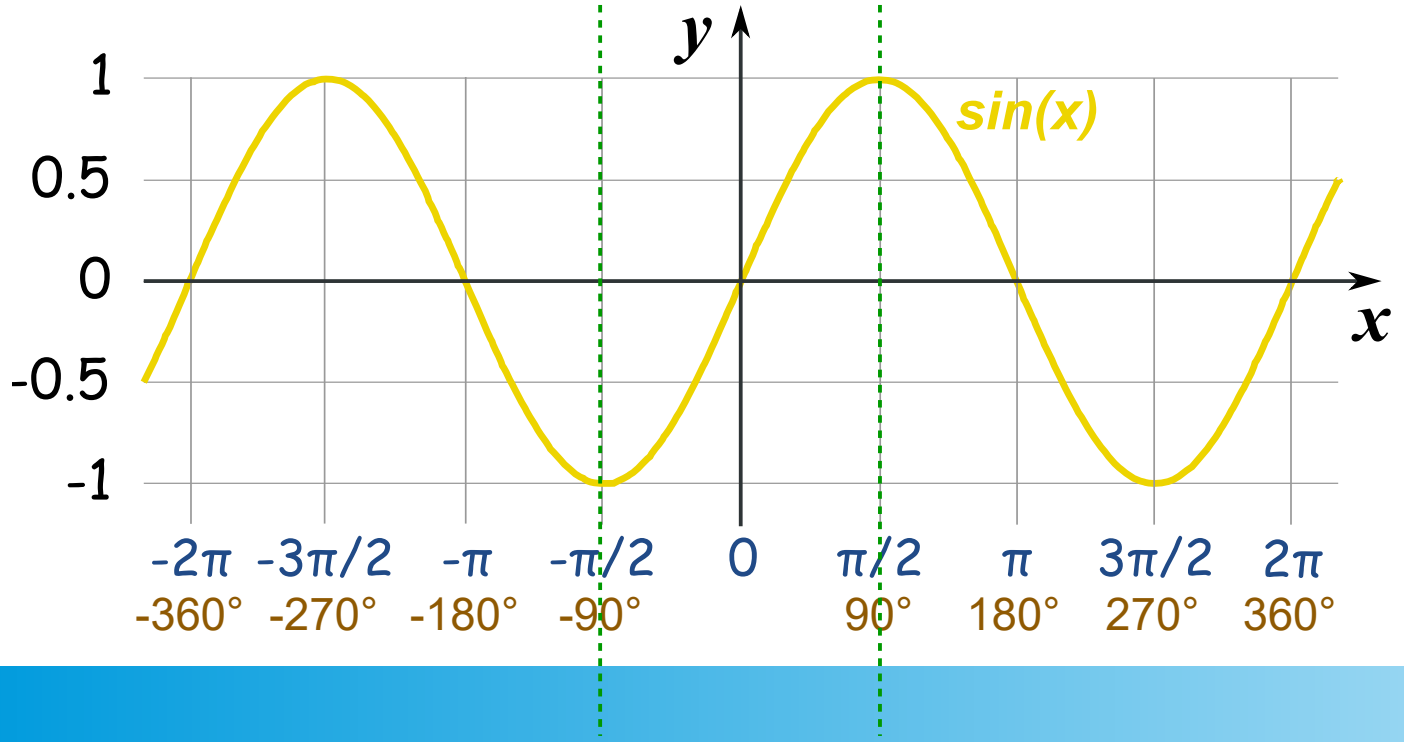
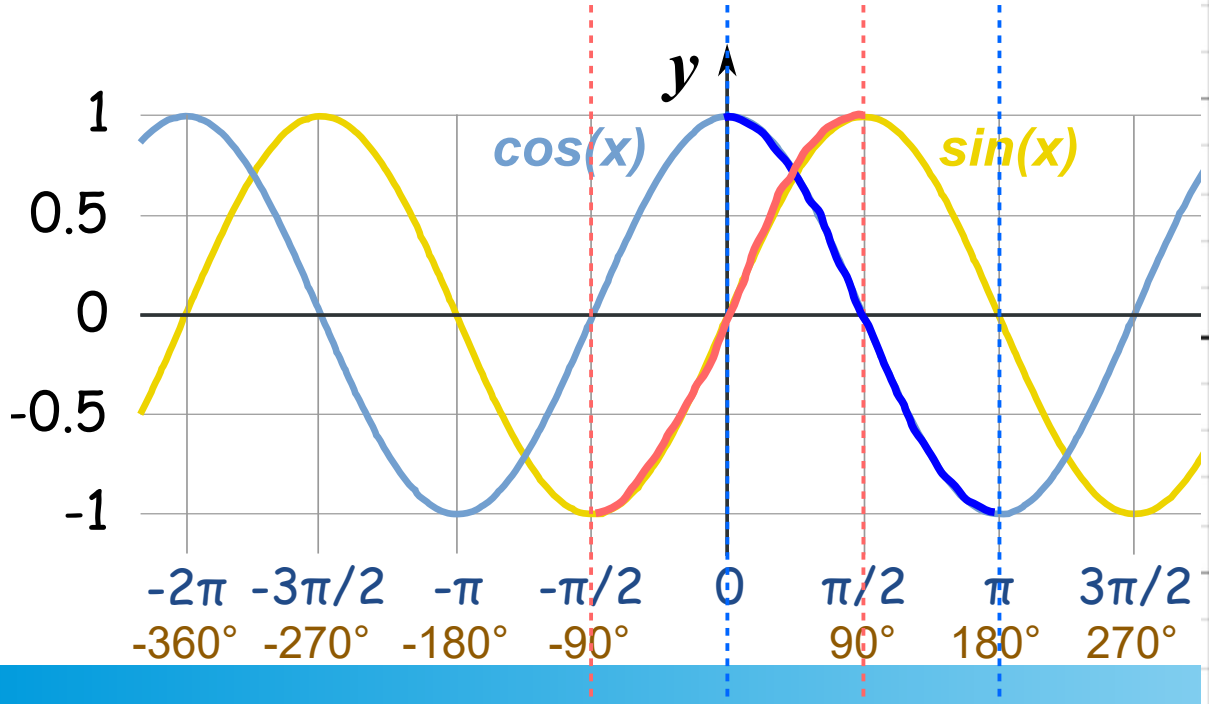


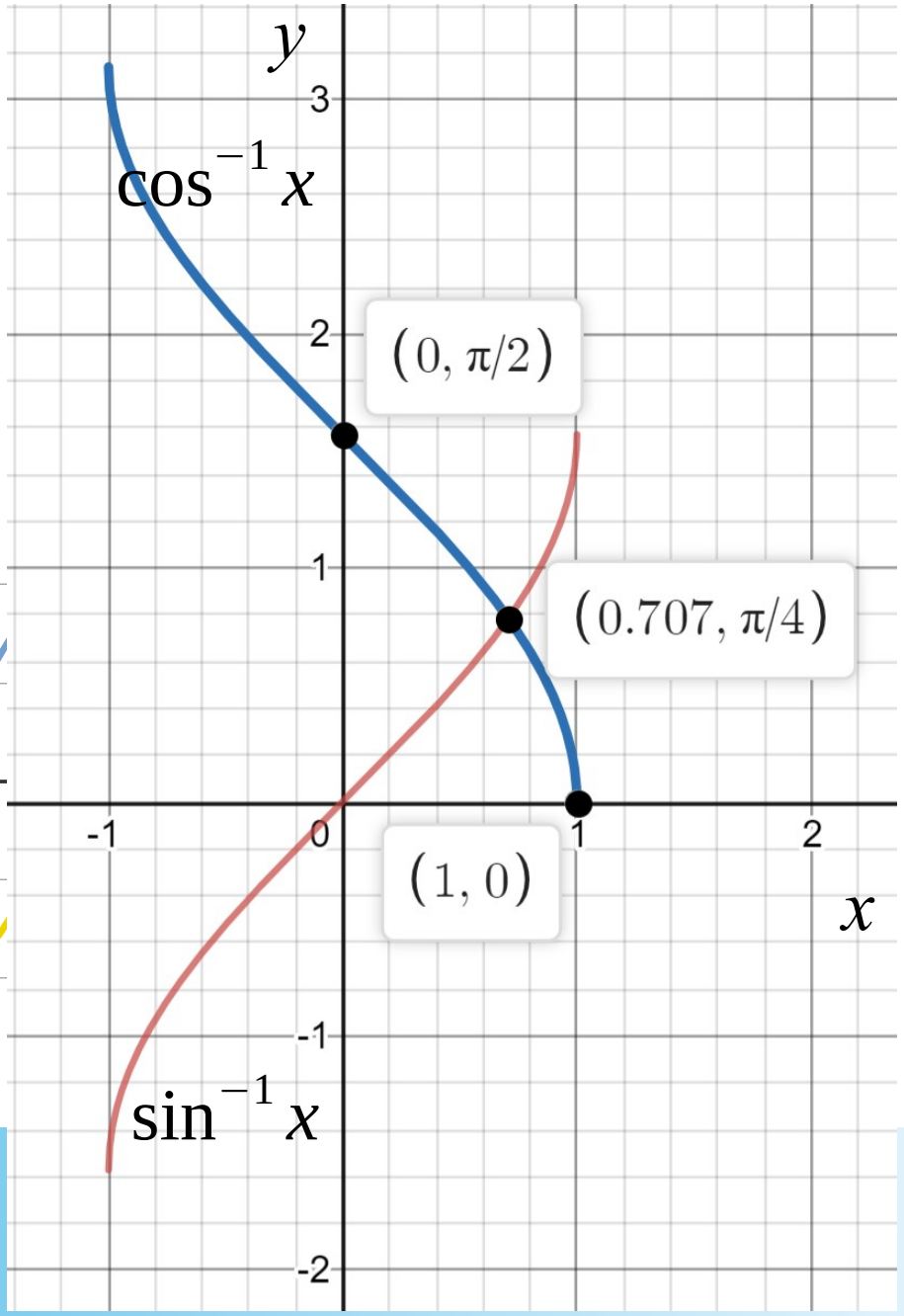
Recall that: *a function is invertible* if it is <u>one-to-one,</u> i.e. there is only one value y for each value x.

The Inverse Trigonometric Functions

$$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$$

$$0 \le \cos^{-1} x \le \pi$$

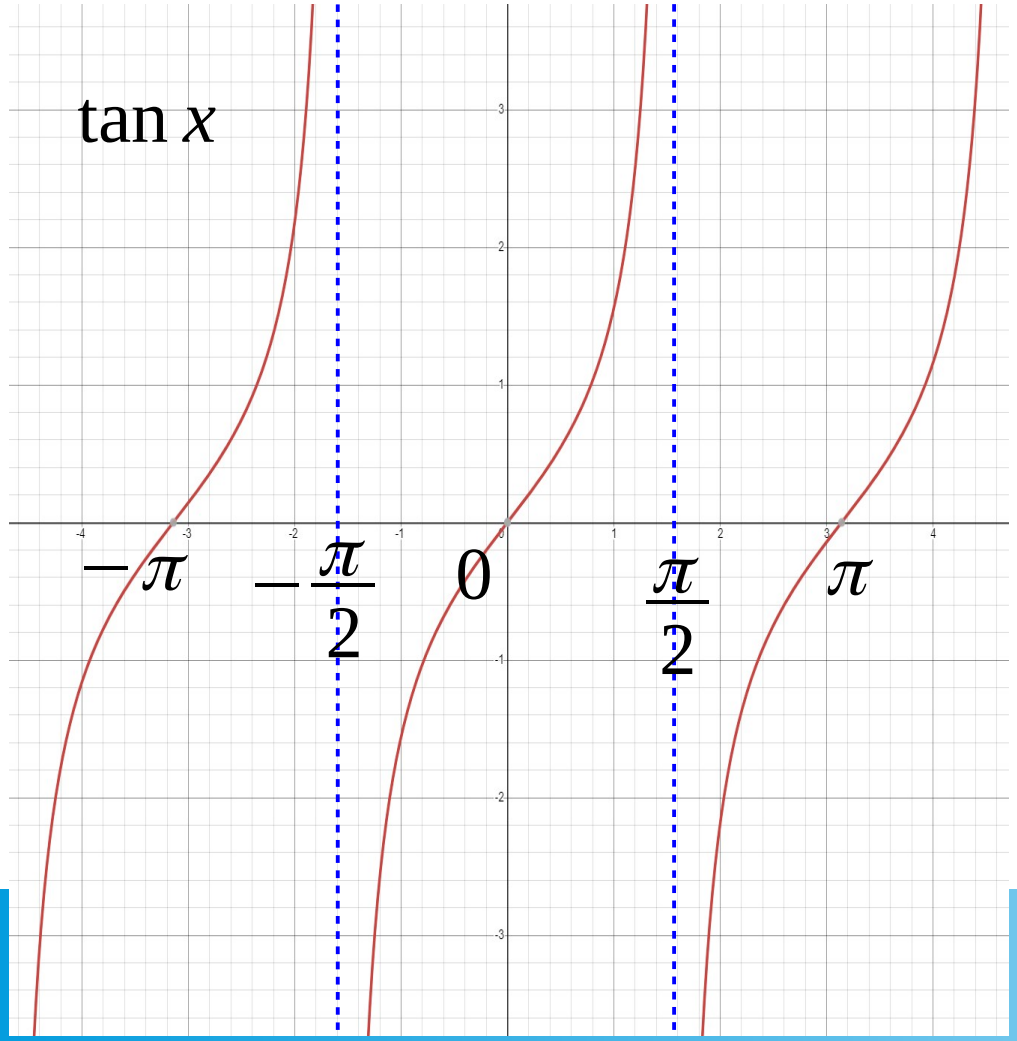


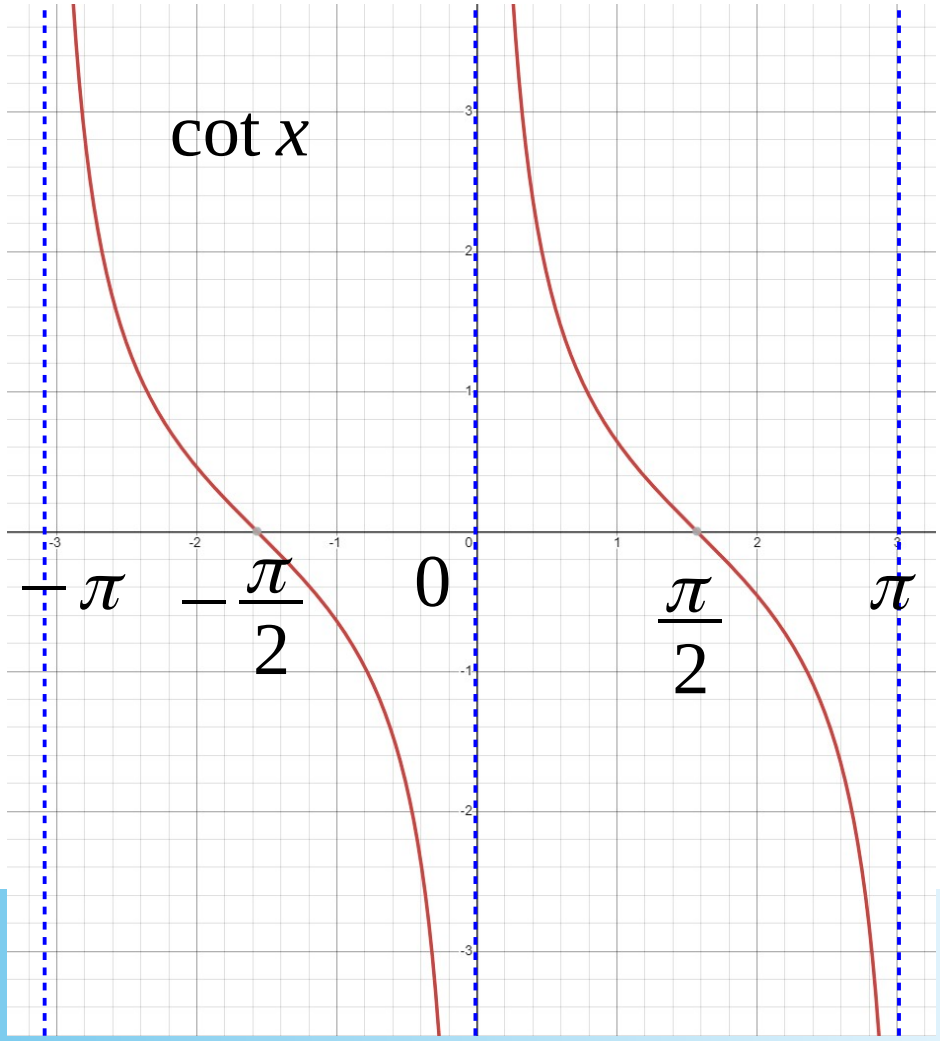


The Inverse Trigonometric Functions

$$-\frac{\pi}{2} < \tan^{-1} y \leq \frac{\pi}{2}$$

$$0 < \cot^{-1} y \leq \pi$$

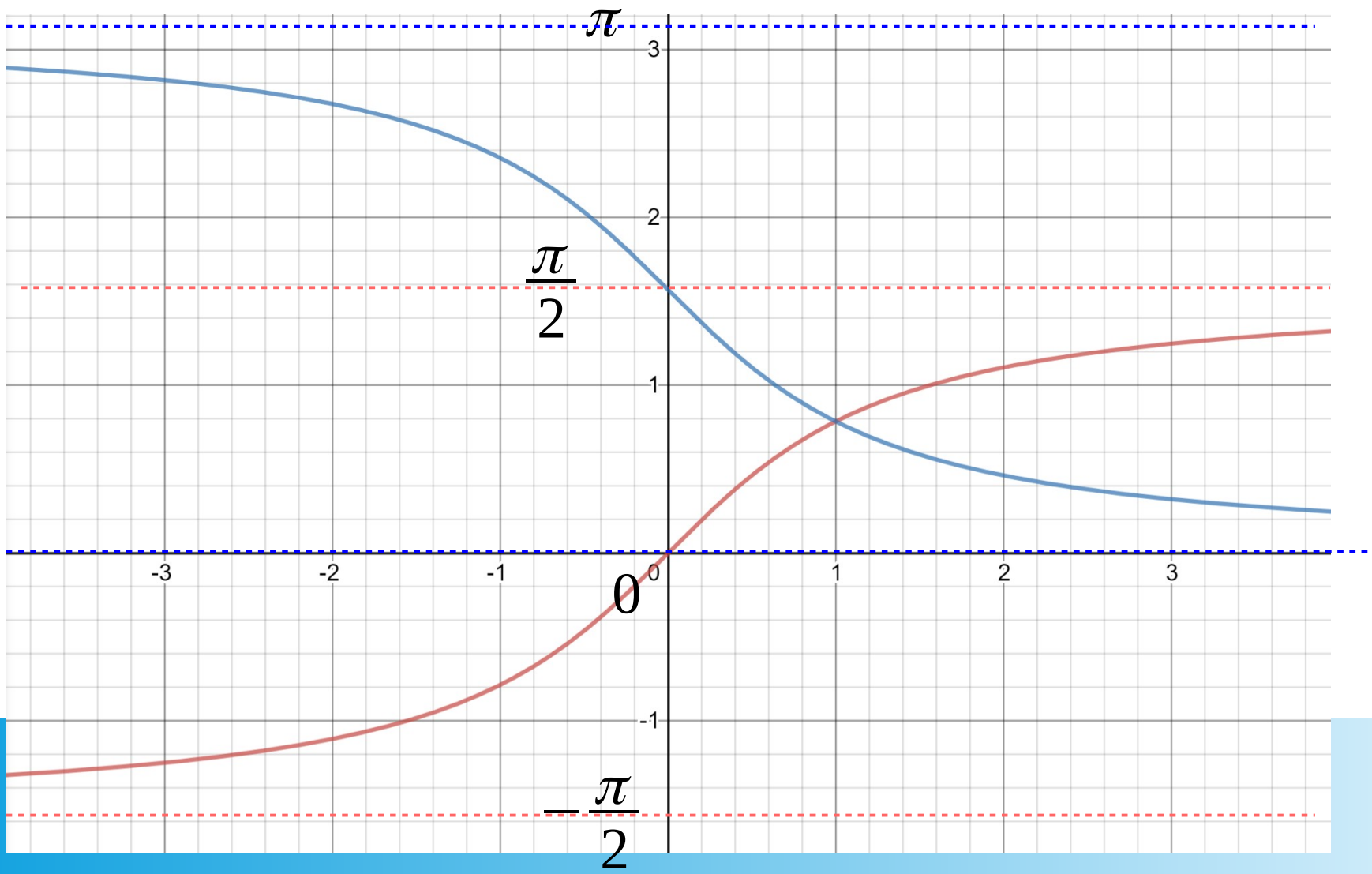




The Inverse Trigonometric Functions

$$-\frac{\pi}{2} < \tan^{-1} x \leq \frac{\pi}{2}$$

$$0 < \cot^{-1} x \leq \pi$$



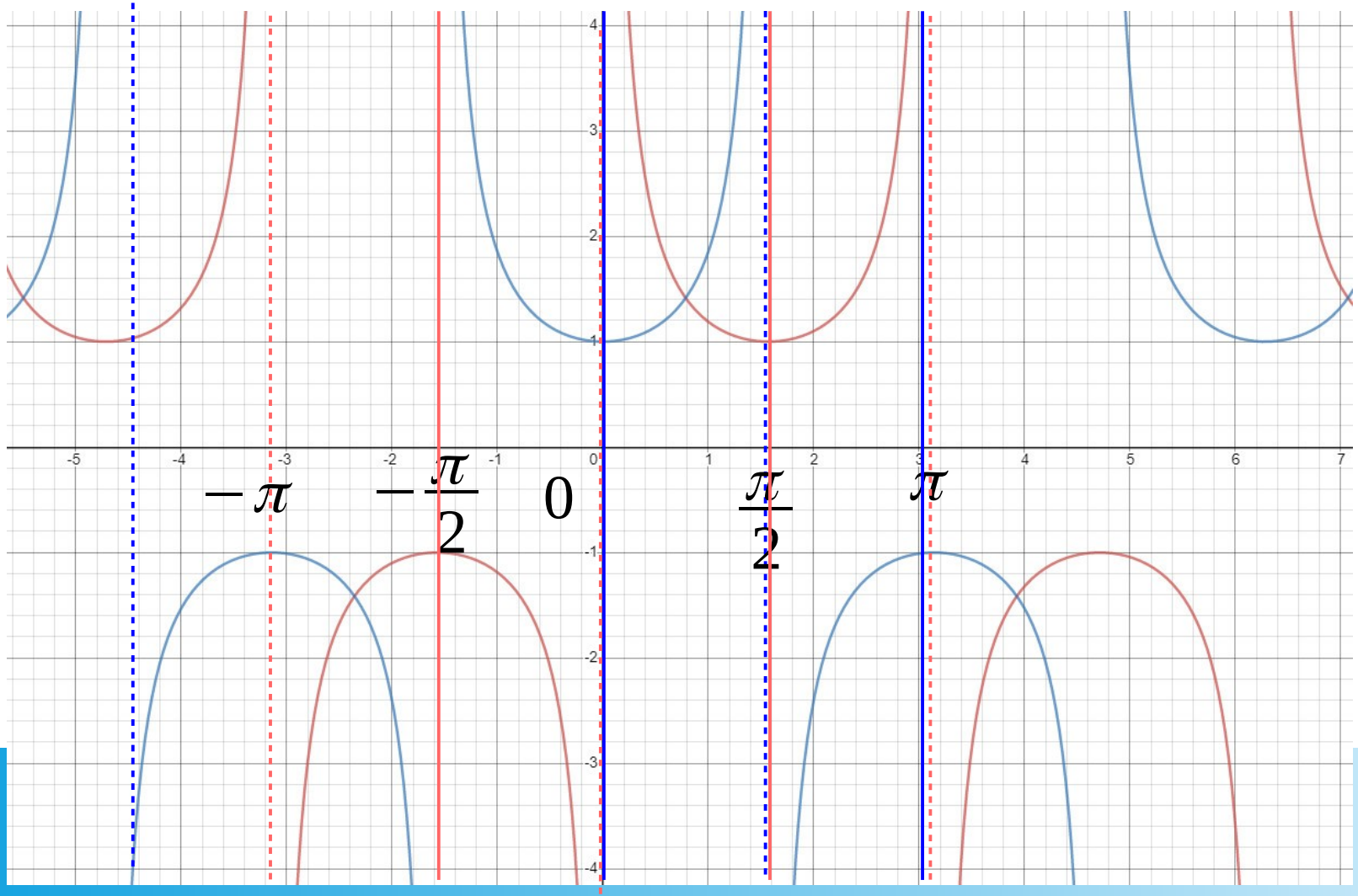
The Inverse Trigonometric Functions

$$-\frac{\pi}{2} \le \csc^{-1} y \le \frac{\pi}{2}$$

$$\csc^{-1} y \neq 0$$

$$0 \le \sec^{-1} y \le \pi$$

$$\sec^{-1} y \neq \frac{\pi}{2}$$



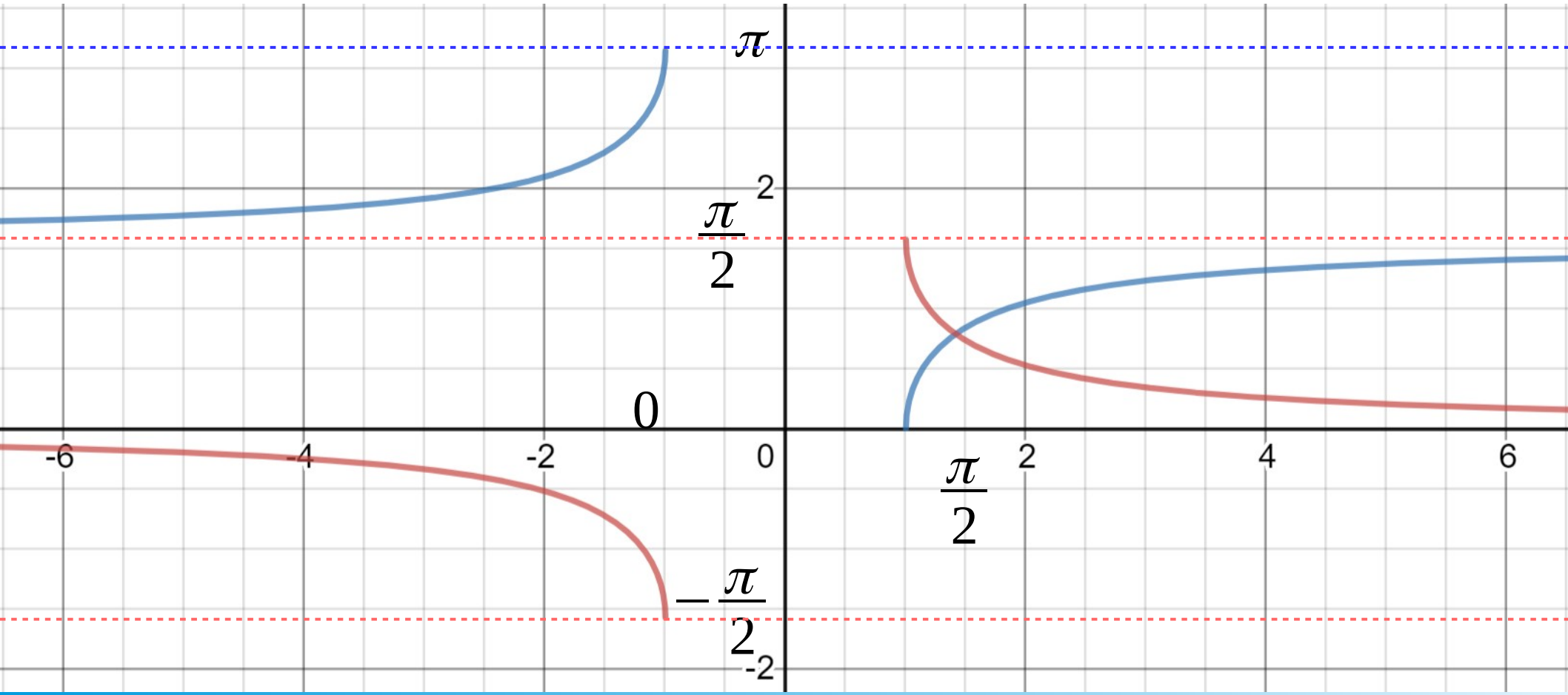
The Inverse Trigonometric Functions

$$-\frac{\pi}{2} \le \csc^{-1} x \le \frac{\pi}{2}$$

$$\csc^{-1} x \neq 0$$

$$0 \le \sec^{-1} x \le \pi$$

$$\sec^{-1} x \neq \frac{\pi}{2}$$



The Inverse Trigonometric Functions

Example 1: Find

(a) $\sin^{-1}\frac{\sqrt{3}}{2}$

(b) $sec^{-1}\sqrt{2}$

(c) $\sin^{-1}-\frac{\sqrt{3}}{2}$

$$-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$$

$$0\le\cos^{-1}x\le\pi$$

$$-\frac{\pi}{2}<\tan^{-1}x\le\frac{\pi}{2}$$

$$0<\cot^{-1}x\le\pi$$

$$-\frac{\pi}{2}\le csc^{-1}x\le\frac{\pi}{2}$$

$$csc^{-1}x\neq 0$$

$$0\le sec^{-1}x\le\pi$$

$$sec^{-1}x\neq\frac{\pi}{2}$$

Values of Trigonometric Functions

θ	0°	$\frac{\pi}{6}$ (30°)	$\frac{5\pi}{6}$ (150°)	$\frac{7\pi}{6}$ (210°)	$\frac{11\pi}{6}$ (330°)	$\frac{\pi}{4}$ (45°)	$\frac{3\pi}{4}$ (135°)	$\frac{5\pi}{4}$ (225°)	$\frac{7\pi}{4}$ (315°)	$\frac{\pi}{3}$ (60°)	$\frac{2\pi}{3}$ (120°)	$\frac{4\pi}{3}$ (240°)	$\frac{5\pi}{3}$ (300°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Sinθ	0	$\frac{1}{2}$		$\frac{-1}{2}$		$\frac{1}{\sqrt{2}}$		$\frac{-1}{\sqrt{2}}$		$\frac{\sqrt{3}}{2}$		$\frac{-\sqrt{3}}{2}$		1	0	-1
θ	0°	$\frac{\pi}{6}$ (30°)	$\frac{11\pi}{6}$ (330°)	$\frac{5\pi}{6}$ (150°)	$\frac{7\pi}{6}$ (210°)	$\frac{\pi}{4}$ (45°)	$\frac{7\pi}{4}$ (315°)	$\frac{3\pi}{4}$ (135°)	$\frac{5\pi}{4}$ (225°)	$\frac{\pi}{3}$ (60°)	$\frac{5\pi}{3}$ (300°)	$\frac{2\pi}{3}$ (120°)	$\frac{4\pi}{3}$ (240°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Cosθ	1	$\frac{\sqrt{3}}{2}$		$\frac{-\sqrt{3}}{2}$		$\frac{1}{\sqrt{2}}$		$\frac{-1}{\sqrt{2}}$		$\frac{1}{2}$		$\frac{-1}{2}$		0	-1	0
θ	0°	$\frac{\pi}{6}$ (30°)	$\frac{7\pi}{6}$ (210°)	$\frac{5\pi}{6}$ (150°)	$\frac{11\pi}{6}$ (330°)	$\frac{\pi}{4}$ (45°)	$\frac{5\pi}{4}$ (225°)	$\frac{3\pi}{4}$ (135°)	$\frac{7\pi}{4}$ (315°)	$\frac{\pi}{3}$ (60°)	$\frac{4\pi}{3}$ (240°)	$\frac{2\pi}{3}$ (120°)	$\frac{5\pi}{3}$ (300°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3\pi}{2}$ (270°)
Tanθ	0	$\frac{1}{\sqrt{3}}$		$\frac{-1}{\sqrt{3}}$		1		-1		√3		-√3		*	0	*
Cotθ	*	√3		-√3		1		-1		$\frac{1}{\sqrt{3}}$		$\frac{-1}{\sqrt{3}}$		0	*	0
θ	0°	30°, 150°		210°, 330°		45°, 135°		225°, 315°		60°, 120°		240°, 300°		90°	180°	270°
Cosecθ	*	2		-2		√2		-√2		$\frac{2}{\sqrt{3}}$		$\frac{-2}{\sqrt{3}}$		1	*	-1
θ	0°	30°, 330°		150°, 240°		45°, 315°		135°, 225°		60°, 300°		120°, 240°		90°	180°	270°
Secθ	1	$\frac{2}{\sqrt{3}}$		$\frac{-2}{\sqrt{3}}$		√2		-√2		2		-2		*	-1	*

The Inverse Trigonometric Functions

Example 2: Evaluate

(a) $\sin\left(\tan^{-1}\sqrt{3}\right)$

(b) $sec\left(\cos^{-1}-0.5\right)$

$$-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$$

$$0\le\cos^{-1}x\le\pi$$

$$-\frac{\pi}{2}<\tan^{-1}x\le\frac{\pi}{2}$$

$$0<\cot^{-1}x\le\pi$$

$$-\frac{\pi}{2}\le csc^{-1}x\le\frac{\pi}{2}$$

$$csc^{-1}x\neq 0$$

$$0\le sec^{-1}x\le\pi$$

$$sec^{-1}x\neq\frac{\pi}{2}$$

The Inverse Trigonometric Functions

Example 3: Evaluate $\sin\left(2\tan^{-1}2\right)$

$$-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$$

$$0\le\cos^{-1}x\le\pi$$

$$-\frac{\pi}{2}<\tan^{-1}x\le\frac{\pi}{2}$$

$$0<\cot^{-1}x\le\pi$$

$$-\frac{\pi}{2}\le csc^{-1}x\le\frac{\pi}{2}$$

$$csc^{-1}x\neq 0$$

$$0\le sec^{-1}x\le\pi$$

$$sec^{-1}x\neq\frac{\pi}{2}$$

The Inverse Trigonometric Functions

Example 4: Find the exact value of x for

$$\tan^{-1} x = \sin^{-1}\left(\frac{2}{5}\right)$$

$$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$$

$$0 \le \cos^{-1} x \le \pi$$

$$-\frac{\pi}{2} < \tan^{-1} x \le \frac{\pi}{2}$$

$$0 < \cot^{-1} x \le \pi$$

$$-\frac{\pi}{2} \le csc^{-1} x \le \frac{\pi}{2}$$

$$csc^{-1} x \ne 0$$

$$0 \le sec^{-1} x \le \pi$$

$$sec^{-1} x \ne \frac{\pi}{2}$$

The Inverse Trigonometric Functions

Example 5: Solve the equation

$$1-y=\cos^{-1}(1-x) \quad \text{for } x.$$

$$-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$$

$$0\le\cos^{-1}x\le\pi$$

$$-\frac{\pi}{2}<\tan^{-1}x\le\frac{\pi}{2}$$

$$0<\cot^{-1}x\le\pi$$

$$-\frac{\pi}{2}\le csc^{-1}x\le\frac{\pi}{2}$$

$$csc^{-1}x\neq 0$$

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