MTH13. p. 539-540/ 12, 16, 28, 30, 34.

N12 Find $\cos 276°$ directly and by using functions of $138°$

Solution: $\cos 276° \approx \boxed{0.105}$ (used calculator)

$\cos 276° = \cos(2\cdot 138°) = \cos 138° \cdot \cos 138° - \sin 138° \cdot \sin 138° \approx$

(15)

$= \cos^2 138° - \sin^2 138° \approx \boxed{0.105}$

N16 Find $\cos 2x$ if $\sin x = -\frac{12}{13}$ (in third quadrant)

Solution: $\cos 2x = 1 - 2\sin^2 \alpha = 1 - 2\cdot\left(-\frac{12}{13}\right)^2 = 1 - 2\cdot \frac{144}{169} =$

(15)

$= \frac{169-288}{169} = -\frac{119}{169}$ or ≈ -0.7

Answer: $\boxed{\cos 2x = -\frac{119}{169} \text{ or } \approx -0.7}$

N28 simplify $\cos^4 u - \sin^4 u$

Solution: $\cos^4 u - \sin^4 u = (\cos^2 u + \sin^2 u)(\cos^2 u - \sin^2 u) =$

difference of squares

$= 1\cdot(\cos^2 u - \sin^2 u) = 1\cdot \cos 2u = \underline{\underline{\cos 2u}}$

(6) (15)

Answer: $\boxed{\cos^4 u - \sin^4 u = \cos 2u}$

N30 simplify $\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x}$

Solution: $\frac{\cos 3x}{\sin x * \cos x}^{*\cos x} + \frac{\sin 3x}{\cos x * \sin x}^{*\sin x} = \frac{\cos 3x \cos x + \sin 3x \sin x}{\cos x \sin x} =$

LCD $= \sin x \cdot \cos x$

$\underset{(12)}{=} \frac{\cos(3x - x)}{\cos x \sin x} = \frac{\cos 2x}{\cos x \sin x} \underset{(14)}{=} \frac{\cos 2x}{\frac{1}{2}\sin 2x} \underset{(5)}{=} 2\cot 2x$

Answer: $\boxed{\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = 2\cot 2x}$

N34 prove identity $2 + \frac{\cos 2\theta}{\sin^2\theta} = \csc^2\theta$

Solution: (Proof)

$$2 + \frac{\cos 2\theta}{\sin^2\theta} = \frac{2\sin^2\theta + \cos 2\theta}{\sin^2\theta} \underset{(15)}{=} \frac{2\sin^2\theta + 1 - 2\sin^2\theta}{\sin^2\theta} =$$

re-write as a fraction with denominator $\sin^2\theta$

$$= \frac{1}{\sin^2\theta} \underset{(1)}{=} \csc^2\theta$$

p. 543 / 8, 10, 22, 28, 32

N8 use half-angle formulas to evaluate $\sin\frac{11\pi}{12}$.

Solution:

$$\sin\frac{11\pi}{12} = \pm\sqrt{\frac{1-\cos\frac{11\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} =$$

use table

$\frac{11\pi}{12}$ is in ~~second~~ quadrant and sin function is positive

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} \text{ or } \approx 0.26$$

Answer: $\boxed{\sin\frac{11\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2} \text{ or } \approx 0.26}$

N10 read description of the problem in the book.

Solution: $\sqrt{\frac{1+\cos 98^\circ}{2}} \underset{(18)}{=} \cos\frac{98^\circ}{2} = \cos 49^\circ \boxed{\approx 0.66}$

if we use calculator to evaluate the radical (originally given), then we will get $\boxed{\approx 0.66}$.

N22 Find the value of $\cos\left(\frac{\alpha}{2}\right)$ if $\sin\alpha = -\frac{4}{5}$ $(180^\circ < \alpha < 270^\circ)$

Solution: 1) let's find $\cos\alpha$: $\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$ from (6)

$$\cos\alpha = -\sqrt{1-\left(-\frac{4}{5}\right)^2} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{25-16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

2) then $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1-3/5}{2}} = -\sqrt{\frac{2}{10}} = -\sqrt{\frac{1}{5}} = \boxed{-\frac{\sqrt{5}}{5}}$

$90^\circ < \alpha/2 < 135^\circ$

$\boxed{\text{or } \approx -0.45}$

MTH13 HW Solutions

p. 543 / 28, 32

$\boxed{N28}$ $\cot\frac{\alpha}{2}$ · in terms of $\sin\alpha$ and $\cos\alpha$

Solution: $\cot\frac{\alpha}{2} = \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = \frac{\pm\sqrt{\frac{1+\cos\alpha}{2}}}{\pm\sqrt{\frac{1-\cos\alpha}{2}}} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$

Answer: $\boxed{\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}}$

$\boxed{N32}$ prove $\cos^2\frac{x}{2}\left[1+\left(\frac{\sin x}{1+\cos x}\right)^2\right] = 1$

Solution: $\cos^2\frac{x}{2}\left(1+\left(\frac{\sin x}{1+\cos x}\right)^2\right) = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2\left(1+\frac{\sin^2 x}{(1+\cos x)^2}\right)$

$= \frac{\cancel{1+\cos x}}{2}\left(\frac{(1+\cos x)^2+\sin^2 x}{(1+\cos x)^{\cancel{2}}}\right) = \frac{1+\cos^2 x+2\cos x+\sin^2 x}{2(1+\cos x)} =$

$= \frac{1+1+2\cos x}{2(1+\cos x)} = \frac{\cancel{2(1+\cos x)}}{\cancel{2(1+\cos x)}} = 1$