MTH 13 HW #14.

p. 379 / 12, 16, 22, 45, 51

(N12) $\log_{12} 122 = \frac{\log 122}{\log 12} \approx \boxed{1.93}$

or

$\log_{12} 122 = \frac{\ln 122}{\ln 12} \approx \boxed{1.93}$

(N16) find $\ln 293 \approx \boxed{5.68}$ or $\approx \boxed{5.7}$

(N22) find $\ln \sqrt{0.000060808} = \frac{1}{2} \ln 0.000060808 \approx$

using properties of logarithms. ($\log_6 x^n = n \log_6 x$)

$\approx \boxed{-4.85}$ or $\boxed{\approx -4.9}$

(N45) solve for x : $\ln(\log x) = 0$

$\ln(\log x) = 0 \rightsquigarrow$ exp. form $\quad e^0 = \log x$

or

$1 = \log x \rightsquigarrow$ exp. form

$\rightsquigarrow$ exp. form. $\quad 10^1 = x$ therefore,

$\boxed{x = 10}$

(N51) read the description in the book.

summary: $i = \frac{\ln 2}{t}$ $\quad i$ - interest rate, t - time.

Find i, if the account is to double in 8.5 years.

Solution: $i = \frac{\ln 2}{8.5} \approx 0.0815$ or 8.15 %.

Answer: $i \approx 0.0815$ or 8.15 %; or $i \approx 0.08$ or 8 %

p. 382 / 4, 10, 14, 16, 22, 26, 30.

(N4) $3^x = \frac{1}{81}$ $\quad 3^x = \frac{1}{3^4}$ $\quad 3^x = 3^{-4}$ therefore, $\boxed{x = -4}$

or

$\log(3^x) = \log\left(\frac{1}{81}\right)$ $\quad x \log 3 = -\log 81$ $\quad x = \frac{-\log 81}{\log 3} = -4$

(N10) $5^{x-1} = 0.07$ - take log or ln of ~~both~~ sides:

$\log 5^{x-1} = \log 0.07$ - use logarithm's property $\log_b x^n = n \log_b x$

$(x-1)\log 5 = \log 0.07$.

$x - 1 = \frac{\log 0.07}{\log 5}$ $\quad x = \frac{\log 0.07}{\log 5} + 1 \approx -0.65$

$\boxed{x \approx -0.65}$

(N14) $15.6^{x+2} = 23^x$ - take log or ln of both sides.

$\ln 15.6^{x+2} = \ln 23^x$

$(x+2)\ln 15.6 = x \ln 23$ $\quad x \ln 15.6 + 2 \ln 15.6 = x \ln 23$

$x \ln 15.6 + 2 \ln 15.6 - x \ln 23 = 0$

$x(\ln 15.6 - \ln 23) = -2 \ln 15.6$

$x = \frac{-2 \ln 15.6}{\ln 15.6 - \ln 23} \approx 14.15$ or 14.2

Answer: $\boxed{x = 14.15}$ or $\boxed{x = 14.2.}$

(N16) $5 \log_{32} x = -3$

$\log_{32} x^5 = -3 \xrightarrow{\text{exp. form}} 32^{-3} = x^5$ $\quad \boxed{x = \sqrt[5]{\frac{1}{32^3}}}$ $\boxed{= 0.125 = \frac{1}{8}}$

or $32 = 2^5$ therefore $32^{-3} = \frac{1}{32^3} = \frac{1}{(2^5)^3} = \frac{1}{2^{15}}$

$\sqrt[5]{\frac{1}{2^{15}}} = \frac{1}{2^3} = \frac{1}{8} = 0.125$

yet another way to solve (3rd):

$5\log_{32} x = -3$ $\quad 5\dfrac{\log x}{\log 32} = -3$

$\log x = \dfrac{-3\cdot\log 32}{5}$

$\Downarrow$ exp. form

$10^{\frac{-3\log 32}{5}} = x$, $\boxed{x = 0.125}$

or $\log_{32} x = -\frac{3}{5}$

$\Downarrow$

$32^{-\frac{3}{5}} = x$

$\boxed{x = 0.125}$ or $\boxed{x = \frac{1}{8}}$

№22 $3\log(2x-1) = 1$

$\log(2x-1) = \frac{1}{3}$ $\xrightarrow{\text{exp form}}$ $10^{\frac{1}{3}} = 2x-1$

$\boxed{x = \dfrac{10^{\frac{1}{3}}+1}{2} \approx 1.58}$. or simply $\boxed{x \approx 1.58}$

№26 $\log_2 x + \log_2(x+2) = 3$ - join the logs, $3 = \log_2 8$

$\log_2 x(x+2) = \log_2 8$

$x(x+2) = 8$ $\quad x^2+2x-8=0$ $\quad (x+4)(x-2)=0$

$x+4=0$ or $x-2=0$

$\log_2(-4)$ is undefined $\rightarrow$ ~~$x=-4$~~ $\quad \boxed{x=2}$

Answer: $\boxed{x = 2}$

№30 $\ln(2x-1) - 2\ln 4 = 3\ln 2$

$\ln(2x-1) = \ln 4^2 = \ln 2^3$

$\ln\left(\dfrac{2x-1}{16}\right) = \ln 8$

$\dfrac{2x-1}{16} = 8$

$2x-1 = 8\cdot 16$

$x = \dfrac{128+1}{2} = 64.5$

Answer: $\boxed{x = 64.5}$