

**№ 10** Evaluate the determinant  $\begin{vmatrix} -20 & -15 \\ -8 & -6 \end{vmatrix}$

**Solution:**  $\begin{vmatrix} -20 & -15 \\ -8 & -6 \end{vmatrix} = (-20)(-6) - (-8)(-15) = 120 - 120 = 0$

**Answer:**  $\begin{vmatrix} -20 & -15 \\ -8 & -6 \end{vmatrix} = 0$

**№ 26** Solve the system of equations by determinants

$$\begin{cases} 3x - y = 5 \\ 3y - 9x = -15 \end{cases}$$

**Solution:** First, let's move terms around to place  $x$ 's under  $x$ 's and  $y$ 's under  $y$ 's:  $\begin{cases} 3x - y = 5 \\ -9x + 3y = -15 \end{cases}$

Then let's find the determinant of the variables coefficients:

$$\begin{vmatrix} 3 & -1 \\ -9 & 3 \end{vmatrix} = 3 \cdot 3 - (-9) \cdot (-1) = 9 - 9 = 0$$

Note that later on we will be dividing by zero then .... but division by zero is undefined!!!!

It must be one of the special cases – either two lines are parallel and don't intersect, or they are the same straight line. Which case is it?

Let's find the other two determinants:

for  $x$  we will need to evaluate the determinant  $\begin{vmatrix} 5 & -1 \\ -15 & 3 \end{vmatrix} = 5 \cdot 3 - (-15) \cdot (-1) = 15 - (15) = 0$   
(the first column is replaced by the constants):

for  $y$  we will need to evaluate the determinant  $\begin{vmatrix} 3 & 5 \\ -9 & -15 \end{vmatrix} = 3 \cdot (-15) - (-9) \cdot 5 = -45 - (-45) = 0$   
(the second column is replaced by the constants):

Both of them are zeros as well!

It means that the two equations represent the same straight line.

**Answer:** There are infinitely many solutions. Dependent system.

Indeed, we can grab the first equation:  $3x - y = 5$  and multiply both sides by  $(-3)$ :  $-9x + 3y = -15$   
- it is the second equation!

**№ 34** Solve the system of equations by determinants. All numbers are approximate.

$$\begin{cases} 0.25d + 0.63n & = & -0.37 \\ -0.61d - 1.80n & = & 0.55 \end{cases}$$

**Solution:**

Let's find the determinant of the variables coefficients:

$$\begin{vmatrix} 0.25 & 0.63 \\ -0.61 & -1.80 \end{vmatrix} = 0.25 \cdot (-1.80) - (-0.61) \cdot (0.63) = -0.45 + 0.3843 = -0.0657$$

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} -0.37 & 0.63 \\ 0.55 & -1.80 \end{vmatrix}}{\begin{vmatrix} 0.25 & 0.63 \\ -0.61 & -1.80 \end{vmatrix}} = \frac{(-0.37)(-1.80) - 0.55 \cdot 0.63}{-0.0657} = \frac{0.3195}{-0.0657} \approx -4.8630$$

$$y = \frac{\begin{vmatrix} 0.25 & -0.37 \\ -0.61 & 0.55 \end{vmatrix}}{\begin{vmatrix} 0.25 & 0.63 \\ -0.61 & -1.80 \end{vmatrix}} = \frac{0.25 \cdot 0.55 - (-0.61)(-0.37)}{-0.0657} = \frac{-0.0882}{-0.0657} \approx 1.3425$$

**Answer:** (-4.8630, 1.3425).