MTH13 Chapter 3 Review

(1) radius increases at the rate of 2 m/s

$A = \pi r^2$ area of a circle w/ radius r.

$r = 2t$

$A = \pi r^2 = \pi (2t)^2 = 4\pi t^2$

Answer: $A = 4\pi t^2$

(2) $f(-4) = 3\cdot(-4)^2 - 2\cdot(-4) + 4 =$

$= 3\cdot 16 + 8 + 4 = 60$

$\boxed{f(-4) = 60}$

$f(x-4) = 3\cdot(x-4)^2 - 2\cdot(x-4) + 4 =$

$= 3(x^2 - 8x + 16) - 2(x-4) + 4 =$

$= 3x^2 - 24x + 48 - 2x + 8 + 4 =$

$= 3x^2 - 26x + 60$

$\boxed{f(x-4) = 3x^2 - 26x + 60}$

MTH 13

Chapter 3 Review

2 $f(-4) = \frac{8}{-4} - 2(-4)^2 = -2 - 2\cdot 16 = -2-32 = -34$

$\boxed{f(-4) = -34}$

$f(x-4) = \frac{8}{x-4} - 2(x-4)^2 = \frac{8}{x-4} - 2(x^2-8x+16) =$

$= \frac{8}{x-4} - 2x^2 + 16x - 32$

$$\boxed{f(x-4) = \frac{8}{x-4} - 2x^2 + 16x - 32}$$

3 $f(x+h) = 3(x+h)^2 - 2(x+h) + 4 =$

$= 3(x^2+2xh+h^2) - 2(x+h) + 4 =$

$= 3x^2 + 6xh + 3h^2 - 2x - 2h + 4$

$$\frac{f(x+h) - \boxed{f(x)}}{h} = \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 4 - \boxed{(3x^2 - 2x + 4)}}{h} =$$

f(x)

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 2x} - 2h \cancel{+4} \cancel{-3x^2} \cancel{+2x} \cancel{-4}}{h} = \frac{6xh + 3h^2 - 2h}{h} =$$

$= 6x + 3h - 2$. So $\boxed{\frac{f(x+h) - f(x)}{h} = 6x + 3h - 2}$

(4)

(a) $f(x) = x^4 + 1$

$f(x)$ is defined for any real number (x), therefore domain: $\mathbb{R}$ (all real numbers)

Let's look at all the possible values of x^4+1:

if $x=0$, $0^4+1 = 1$

if $x=-1$, $(-1)^4+1=2$

if $x=1$, $(1)^2+1=2$

Can I get a value <1? No, because power 4 eliminates - sign.

Therefore, range: all real numbers ≥ 1

(b) $G(z) = -\frac{4}{z^3}$

domain: when $z=0$ we get division by 0, so exclude 0 from the domain.
Other than that it looks that $G(z)$ is defined on all values (except 0)

domain: all real numbers except 0

range: Can I get 0? (from $-\frac{4}{z^3}$) No, as $|z|$ grows, we will be getting closer to 0, but we will never reach it!

if $z = 1$ we get $-\frac{4}{(+1)^3} = -4$

if $z = -1$ we get $-\frac{4}{(-1)^3} = 4$

and so forth.

It seems that we can get any value, for $-\frac{4}{z^3}$, except for 0.

range: all real numbers except 0.

(c) $g(t) = \frac{8}{\sqrt{t+4}}$.

domain: 1) $t + 4 \geq 0$, otherwise $\sqrt{\ }$ is undefined!

$t \geq -4$

2) $t + 4 \neq 0$, otherwise we get division by 0!

$t \neq -4$

Therefore, the domain: all real numbers > -4

range: $8 > 0$ and $\sqrt{t+4} > 0$, therefore we can only get positive values from $\frac{8}{\sqrt{t+4}}$.

range: all positive real numbers.

(d) $f(x) = \sqrt{6-x}$

domain: $\sqrt{6-x}$ is defined when $6 - x \geq 0$

Therefore, the domain is all real numbers ≤ 6.

range: $\sqrt{6-x}$ produces only positive real numbers, and 0.

$\sqrt{6-6} = 0$

$\sqrt{6-0} = \sqrt{6}$

$\sqrt{6-(-3)} = \sqrt{9} = 3$

...

Therefore, the range is all non-negative real numbers.

(e) $F(y) = 1 - 2\sqrt{y}$

domain: $\sqrt{y}$ is defined when $y \geq 0$

No other "problems"

domain: all real numbers ≥ 0

range: $\sqrt{y} \geq 0$, therefore $1 - 2\sqrt{y} \leq 1$

y	$1-2\sqrt{y}$
0	1
1	$1-2\sqrt{1} = 1-2 = -1$
4	$1-2\sqrt{4} = 1-2\cdot 2 = -3$
....	

range: all real numbers ≤ 1

(f) $f(n) = 1 + \frac{2}{(n-5)^2}$

domain: when $n=5$ we get division by 0
- exclude 5 from the domain.

Other than that everything is fine $\left(1+\frac{2}{(n-5)^2}\right)$ is defined.

domain: all real numbers except 5.

range: $1 + \frac{2}{(n-5)^2}$

← positive

positive

positive

← always positive

← never 0.

1 + positive number, hence $1+\frac{2}{(n-5)^2} > 1$

Therefore, the range is all real numbers > 1

(g) $F(x) = 3 - |x|$

domain: all real numbers (no problems, i.e. where $3-|x|$ is undefined)

range: $|x|$ is always positive

x	$3-\|x\|$
0	$3-0=3$
1	$3-1=2$
-1	$3-\|-1\|=3-1=2$
2	$3-2=1$

← values are decreasing.

3 is max.

i.e. $3-|x| \le 3$

Therefore, the range is all real numbers ≤ 3

5

a) $y = x^2 - 8x - 5$ is a parabola, $\quad y = ax^2 + bx + c$

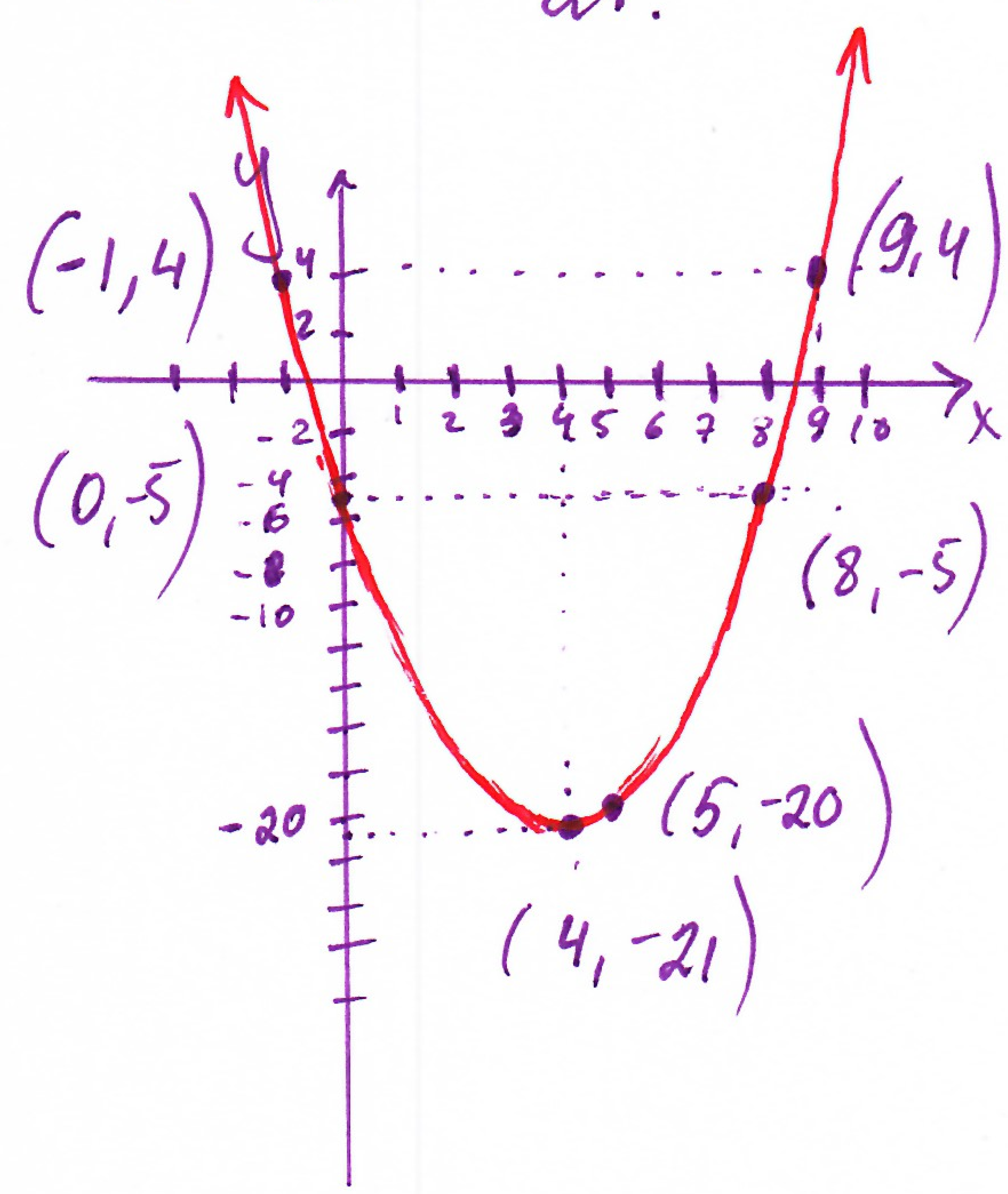
since the leading coefficient $a = 1 > 0$,

it is cupped upwards.

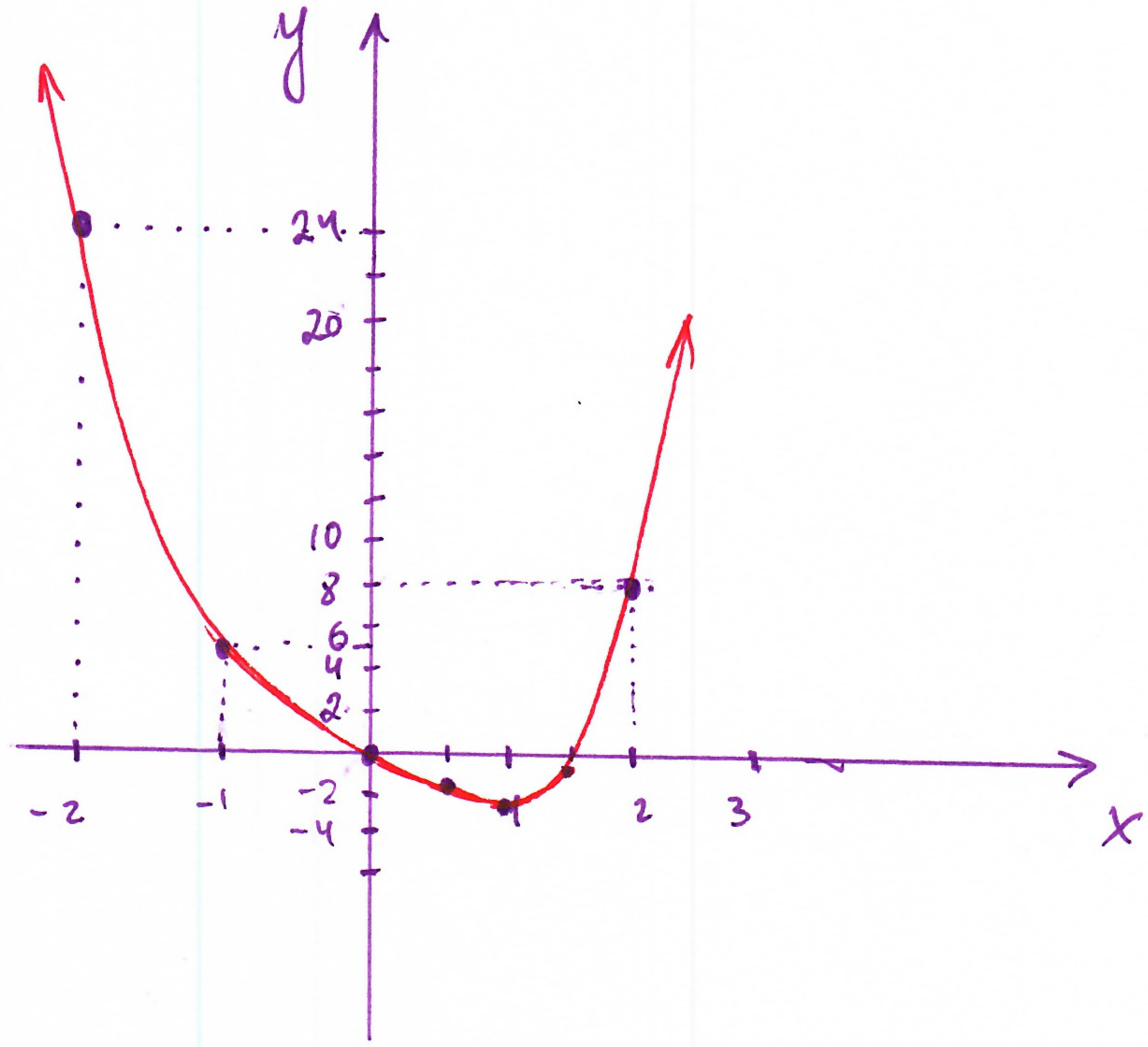
vertex: x-coord. $= -\frac{b}{2a} = -\frac{-8}{2 \cdot 1} = 4.$

at 4 we get: $4^2 - 8 \cdot 4 - 5 = 16 - 32 - 5 = -21.$
(x = 4)

x	y	
0	$0^2 - 8 \cdot 0 - 5 = -5$	(0, -5)
-2	$(-2)^2 - 8 \cdot (-2) - 5 = 15$	(-2, 15)
-1	$(-1)^2 - 8 \cdot (-1) - 5 = 4$	(-1, 4)
5	$5^2 - 8 \cdot 5 - 5 = -20$	(5, -20)
9	$9^2 - 8 \cdot 9 - 5 = 4$	(9, 4)



(b) $y = x^4 - 4x$



x	y
0	$0^4 - 4\cdot 0 = 0$
1	$1^4 - 4\cdot 1 = -3$
2	$2^4 - 4\cdot 2 = 8$
3	$3^4 - 4\cdot 3 = 69$
grows fast !	
-1	$(-1)^4 - 4\cdot(-1) = 5$
-2	$(-2)^4 - 4\cdot(-2) = 24$

grows fast !

we need to see what happens between $x = 0, 1, 2$

x	y
$\frac{1}{2}$	$\left(\frac{1}{2}\right)^4 - 4\cdot\frac{1}{2} = -1.9377$
$\frac{3}{2}$	$\left(\frac{3}{2}\right)^4 - 4\cdot\frac{3}{2} = -0.9375$

(c) $y = \frac{x}{x+1}$ it is hyperbola

note that at $x = -1$, $\frac{x}{x+1}$ is undefined!

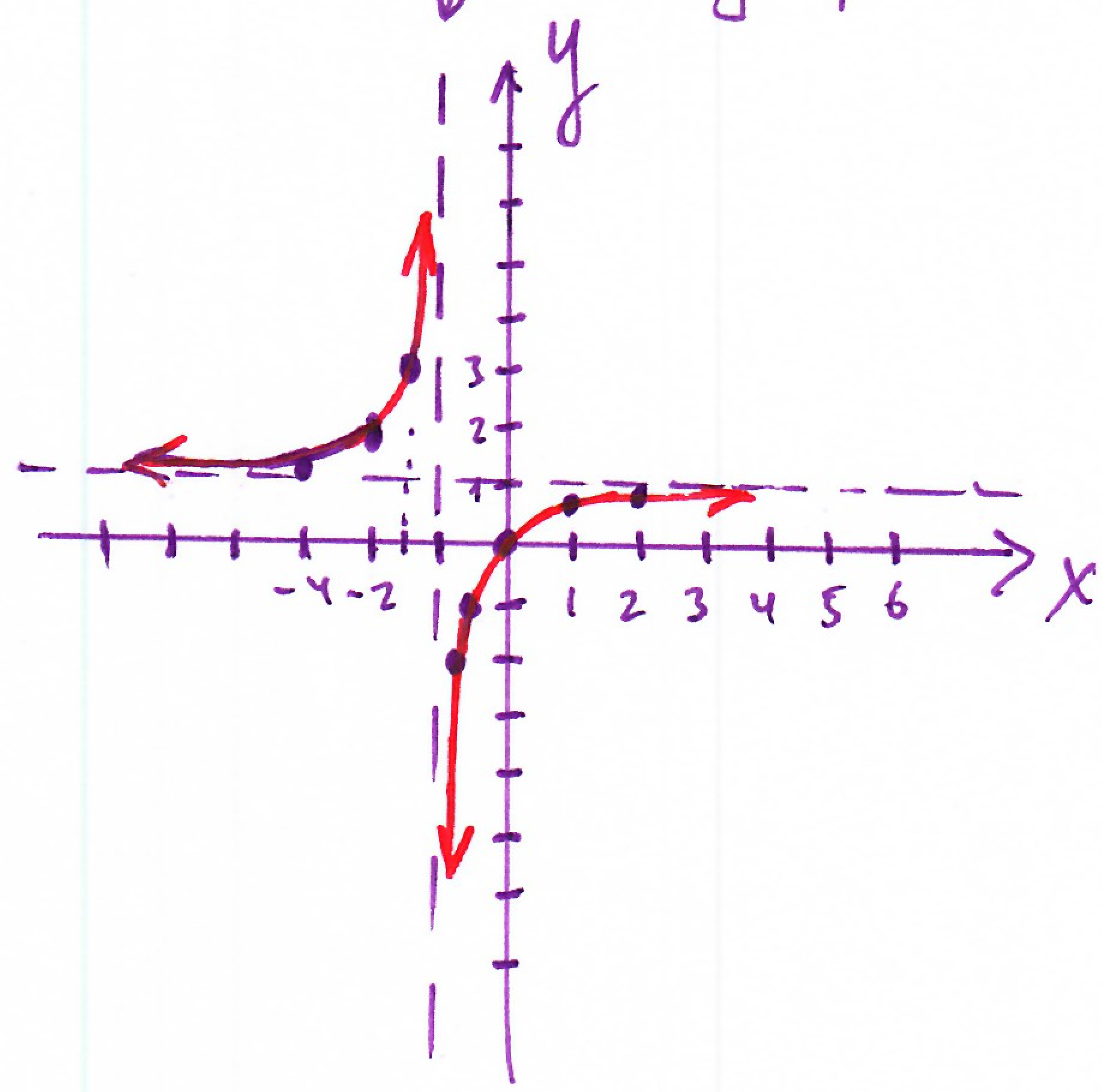
- vertical asymptote

x	$\frac{x}{x+1}$	
0	$\frac{0}{0+1} = 0$	$(0,0)$
1	$\frac{1}{1+1} = \frac{1}{2}$	$(1, \frac{1}{2})$
2	$\frac{2}{2+1} = \frac{2}{3}$	$(2, \frac{2}{3})$
-2	$\frac{-2}{-2+1} = +2$	$(-2, +2)$
-3	$\frac{-3}{-3+1} = \frac{3}{2}$	$(-3, \frac{3}{2})$
-4	$\frac{-4}{-4+1} = \frac{4}{3}$	$(-3, \frac{4}{3})$

approaching 1, but will never touch it

$y = 1$ horizontal asymptote

$x = -1$ vertical asymptote



let's see what happens between 0 and -1, and -1 and -2

x	y
$-\frac{1}{2}$	$\frac{-\frac{1}{2}}{-\frac{1}{2}+1} = -1$
$-\frac{2}{3}$	$\frac{-\frac{2}{3}}{-\frac{2}{3}+1} = -2$
$-\frac{3}{2}$	$\frac{-\frac{3}{2}}{-\frac{3}{2}+1} = 3$

(d) $f(x) = \sqrt{4+2x}$

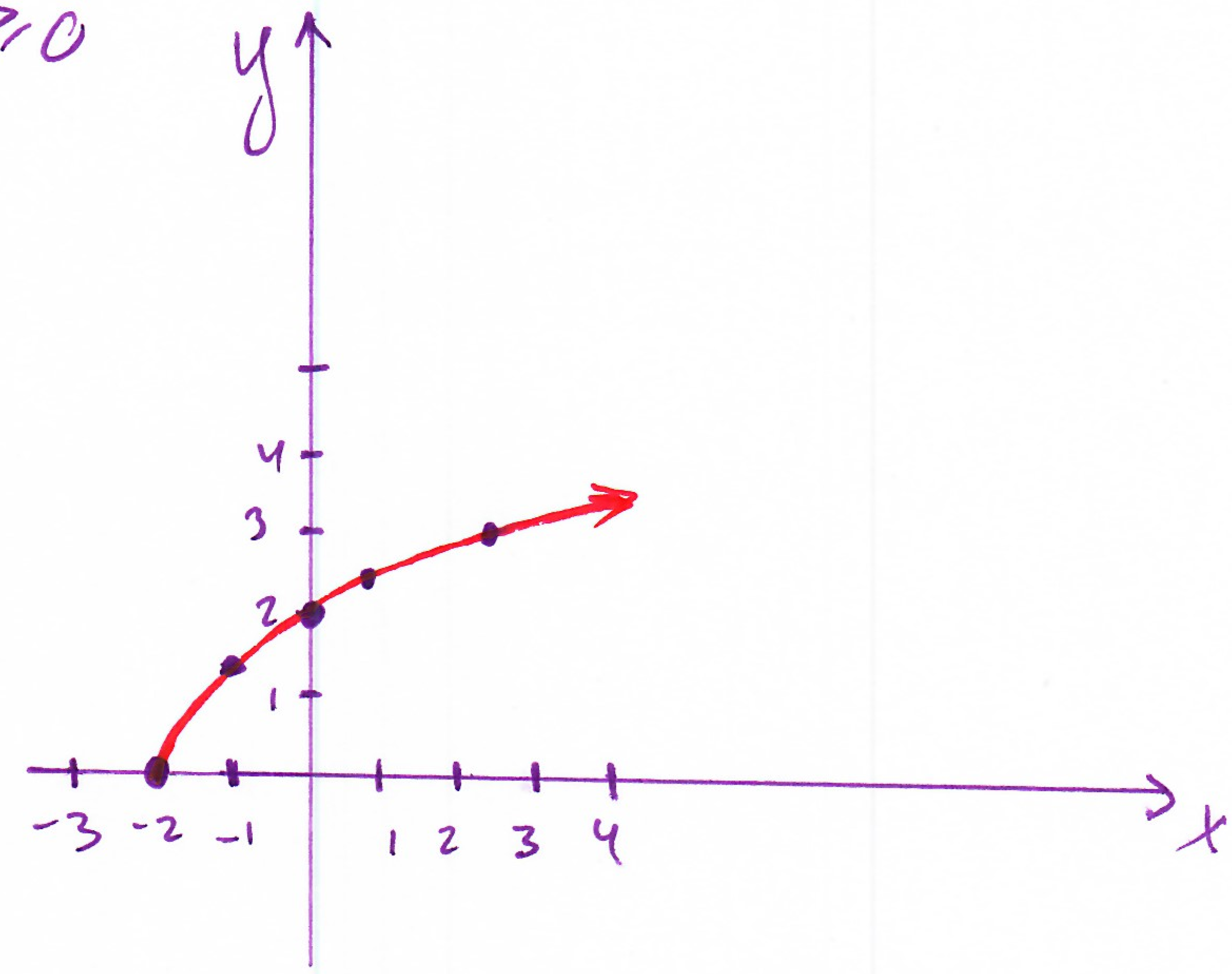
note that $\sqrt{4+2x} \geq 0$

and $4+2x$ must be ≥ 0

$4+2x \geq 0$

$x \geq -2$

x	y
-2	$\sqrt{4+2\cdot(-2)} = \sqrt{0} = 0$
-1	$\sqrt{4+2(-1)} = \sqrt{2} \approx 1.4$
0	$\sqrt{4} = 2$
1	$\sqrt{4+2\cdot 1} = \sqrt{6} \approx 2.4$
$\frac{5}{2}$	$\sqrt{4+2\cdot\frac{5}{2}} = \sqrt{9} = 3$



Chapter 13 Review

1 (a) $\log \frac{4}{5} = \log 4 - \log 5$ by the rule $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\neq \dfrac{\log 4}{\log 5}$

(no such rule)

False

we can evaluate both sides, and compare the results as well :

$\log \frac{4}{5} \approx -0.09691$ $\qquad \dfrac{\log 4}{\log 5} \approx 0.86135$

not the same .

(b) $\log \frac{1}{100} = \log 10^{-2} = -2$, hence True

(c) $2 \ln x - 1 = \ln x$

$2 \ln x - \ln x = 1$

$\ln x = 1$

re-writing in exp. form : $e^{1} = x$, so $x = e$.

True

(2) (a) $\log_9 x = 3$

re-write as exponent : $9^3 = x$, so $x = 9 \cdot 9 \cdot 9 = 729$

$\boxed{x = 729}$

(b) $\log_4 \sin x = -0.5$

re-write in exp. form : $4^{-0.5} = \sin x$

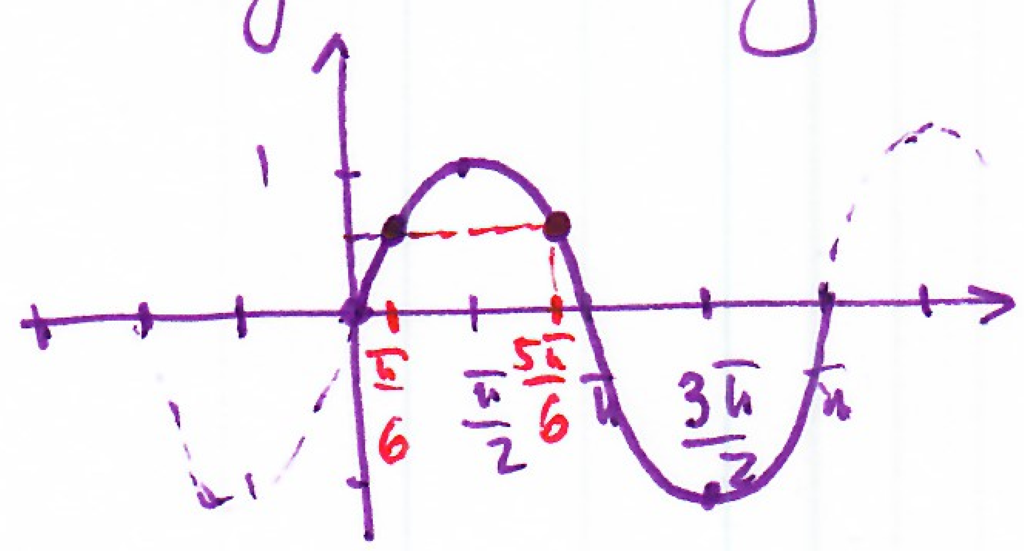
$$4^{-\frac{1}{2}} = \sin x$$

$$\frac{1}{\sqrt{4}} = \sin x$$

So $\sin x = \frac{1}{2}$

Recall trigonometry : $\sin x = \frac{1}{2}$ if $x = 30^\circ \pm \text{period} \cdot k$ (150°; $360^\circ \cdot k$)

or $x = \frac{\pi}{6} \pm 2\pi k$, $\frac{5\pi}{6} \pm 2\pi k, k \in \mathbb{Z}^+$ pos. integer



$\boxed{x = \frac{\pi}{6} \pm 2\pi k, \; \frac{5\pi}{6} \pm 2\pi k, \; k \text{ is positive integer.}}$

(c) $\log_9 27 = x$

✗ re-writing as exponent : $9^x = 27$, x hard to guess,

✓ then $\log_9 27 = \log_9 9 \cdot 3 = \log_9 9 + \log_9 3 = 1 + \log_9 3$

so, $1 + \log_9 3 = x$ $\quad \log_9 3 = x - 1$

(c) continues.

re-write as exponent:

$$9^{x-1} = 3$$

ok, so $x-1$ must be $\frac{1}{2}$ because $\sqrt{9} = 3$ or $9^{\frac{1}{2}} = 3$.

$$x - 1 = \frac{1}{2}$$

$$x = \frac{1}{2} + 1 = \frac{3}{2}.$$

$$\boxed{x = \frac{3}{2} = 1.5}$$

(d) $\log_{12} 144 = x - 3$

re-write in exponential form:

$$12^{x-3} = 144$$

then $x - 3 = 2$ because $12^2 = 144$

so $\boxed{x = 5}$

(e) $\log_x \frac{1}{243} = 5$

re-write in exp. form:

use this instead

$x^5 = \frac{1}{243}$ $\qquad \frac{1}{243} = \frac{1}{3^5} = \cancel{3^{-5}} = \left(\frac{1}{3}\right)^5$

so, $x^5 = \left(\frac{1}{3}\right)^5$ $\quad$ Hence $\boxed{x = \frac{1}{3}}$

(f) $\log_x 8 = \frac{1}{2}$

re-write as exponent: $x^{\frac{1}{2}} = 8$ or $\sqrt{x} = 8$

then $\boxed{x = 64}$

(3)

(a) $\log_3(6x) = \log_3 \overset{2\cdot 3}{6} + \log_3 x = \log_3 2 + \underbrace{\log_3 3}_{=1} + \log_3 x =$

$= \boxed{1 + \log_3 2 + \log_3 x}$

(b) $\log_5\left(\frac{7}{a^3}\right) = \log_5 7 - \log_5 a^3 = \boxed{\log_5 7 - 3\log_5 a}$

(c) $\log_4 \sqrt{48} = \log_4 48^{\frac{1}{2}} = \frac{1}{2}\log_4 \overset{16\cdot 3}{48} = \frac{1}{2}\left(\log_4 16 + \log_4 3\right) =$

$= \frac{1}{2}\left(\underbrace{\log_4 4^2}_{=2} + \log_4 3\right) = \frac{1}{2}\left(2 + \log_4 3\right) = \boxed{1 + \frac{1}{2}\log_4 3}$

(d) $\log \sqrt[4]{32y} = \log (32y)^{\frac{1}{4}} = \frac{1}{4} \log (32y) =$

$= \frac{1}{4}(\log 32 + \log y) = \frac{1}{4} \log \underset{2^5}{32} + \frac{1}{4}\log y =$ don't stop!

$= \frac{1}{4} \log 2^5 + \frac{1}{4} \log y = \frac{1}{4} \cdot 5 \log 2 + \frac{1}{4} \log y =$

$= \boxed{\frac{5}{4} \log 2 + \frac{1}{4} \log y}$

(e) $\log_3\left(\frac{9}{x}\right) = \log_3 9 - \log_3 x = \log_3 3^2 - \log_3 x = \boxed{2 - \log_3 x}$

(f) $\log_{10}(1000x^4) = \log_{10} \underset{10^3}{1000} + \log_{10} x^4 = \log 10^3 + 4 \log x =$

$= \boxed{3 + 4 \log x}$

$\log \equiv \log_{10}$
common logarithm.

(g) $\log_3 (9^2 \times 6^3) = \log_3 9^2 + \log_3 6^3 = 2 \log_3 \underset{3^2}{9} + 3 \log_3 \underset{2 \cdot 3}{6} =$

$= 2\underbrace{\log_3 3^2}_{=2} + 3(\log_3 2 + \underbrace{\log_3 3}_{=1}) =$

$= 4 + 3 \log_3 2 + 3 = \boxed{7 + 3 \log_3 2}$

4

(a) $\ln 6 + 3 \ln e^2 \xrightarrow{(6)} - 9 \ln x \xrightarrow{(6)} =$

$= \ln 6 + \ln (e^2)^3 - \ln x^9 =$

$= \ln 6 + \ln e^6 - \ln x^9 =$

(4) (5)

$= \boxed{\ln \frac{6e^6}{x^9}}$

(1) $\log_b 1 = 0$

(2) $\log_b b = 1$

(3) $\log_b b^n = n$

(4) $\log_b (xy) = \log_b x + \log_b y$

(5) $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

(6) $\log_b x^n = n \log_b x$

(7) $\log_b x = \frac{\log_a x}{\log_a b}$

(b) $2(\log_9 y + 2 \log_9 x \xrightarrow{(6)}) - 1 =$

$= 2(\log_9 y + \log_9 x^2) - 1 \underset{(4)}{=} 2 \xrightarrow{(6)} (\log_9 (y \cdot x^2)) - 1 =$

$= \log_9 (y \cdot x^2)^2 - 1 = \log_9 (y^2 x^4) - \log_9 9 \underset{(5)}{=} \boxed{\log_9 \frac{y^2 x^4}{9}}$

$\log_9 9$ by (2)

(c) $\log_7 y - 2 \log_7 5 \xrightarrow{(6)} + \log_7 x + 3 =$

$\log_7 7^3$ by (3)

$= \log_7 y \underset{(5)}{-} \log_7 5^2 \underset{(4)}{+} \log_7 x \underset{(4)}{+} \log_7 7^3 = \boxed{\log \frac{7^3 y x}{5^2}}$

6/10

(d) $\overset{(6)}{\frac{1}{2}} \log_7 y - \underset{\underset{\log_7 7^5 \text{ by } (3)}{\uparrow}}{5} = \log_7 y^{\frac{1}{2}} - \underset{(5)}{\log_7 7^5} = \boxed{\log \frac{y^{\frac{1}{2}}}{7^5} = \log \frac{\sqrt{y}}{7^5}}$

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(6)

(a) $\log_3 0.0303 = \dfrac{\log 0.0303}{\log 3} \approx -3.18$

(b) $\log_{17} 645 = \dfrac{\ln 645}{\ln 17} \approx 2.28$

(c) $\log_5 0.5 = \dfrac{\log 0.5}{\log 5} \approx -0.43$

(7) $e^{2x} = 5$

apply ln to both sides:

$\ln e^{2x} = \ln 5$

$2x = \ln 5 \qquad x = \dfrac{\ln 5}{2} \approx 0.805$

(b) $\dfrac{2 \cdot 5^x}{2} = \dfrac{15}{2}$

$5^x = 7.5$ apply log to both sides:

$\log 5^{x} = \log 7.5$

$x \cdot \log 5 = \log 7.5$

$x = \dfrac{\log 7.5}{\log 5} \approx 1.252.$

(c) $\dfrac{2^x}{3^{1-x}} = 12^{x+1}$ apply ln to both sides:

$$\ln\left(\frac{2^x}{3^{1-x}}\right) = \ln 12^{x+1}$$

$$\ln 2^{x} - \ln 3^{1-x} = \ln 12^{x+1}$$

$$x \ln 2 - (1-x)\ln 3 = (x+1)\ln 12$$

$$x \ln 2 - \ln 3 + x\ln 3 = x\ln 12 + \ln 12$$

$$x\ln 2 - x\ln 12 + x\ln 3 = \ln 12 + \ln 3$$

$$x(\ln 2 - \ln 12 + \ln 3) = \ln 12 + \ln 3$$

$$x\left(\ln \frac{2\cdot 3}{12}\right) = \ln(12\cdot 3)$$

$$x \cdot \ln \frac{1}{2} = \ln 36$$

$$x = \frac{\ln 36}{\ln \frac{1}{2}} \approx -5.17$$

(d) $\log_8(x+2) = 2 - \log_8 2$ join the right side into one logarithm.

$2 \uparrow \log_8 8^2$

$\log_8(x+2) = \log_8 8^2 - \log_8 2$

$\log_8(x+2) = \log_8 \frac{64}{2}$

$\log_8(x+2) = \log_8 32$

dropping the logarithms: $x+2=32$ $x = 30$

(e) $\log(n+2) + \log n = 1$

$\log((n+2)n) = 1$

re-writing in exp. form: $10^1 = n(n+2)$

$n^2 + 2n = 10$

$n^2 + 2n - 10 = 0$ solve quadratic equation $(ax^2+bx+c=0)$.

$D = b^2 - 4ac = 2^2 - 4\cdot1\cdot(-10) = 4+40 = 44 > 0$, 2 solutions.

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{44}}{2\cdot1} = \frac{-2\pm2\sqrt{11}}{2} = -1 \pm \sqrt{11}$

$x_1 = -1 + \sqrt{11} > 0$, OK

~~$x_2 = -1-\sqrt{11} < 0$~~ not a solution!

$\uparrow$ $\log(-1-\sqrt{11})$ is undefined

Answer: $n = -1+\sqrt{11}$

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