**BRONX COMMUNITY COLLEGE**

**of The City University of New York**

**DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE**

**CSI 35 Test 2 Review Questions**

# 1 Relations and Digraphs

1. The domain of relation $R$ is $\{2, 3, 5, 6, 15, 20, 21\}$.

   For $x$, $y$ in the domain, $xRy$ if $y$ is an integer multiple of $x$.

   Draw the arrow diagram for the relation $R$.

2. For the relation $R = \{(a, b)|a \equiv b(\texttt{mod}\ 7)\}$ on the set $\{1, 2, 3, 8, 10, 15\}$

   **a)** List all the ordered pairs in $R$

   **b)** Give matrix representation of relation $R$

   **c)** Give digraph representing relation $R$

   **d)** Determine whether relation $R$ is reflexive, symmetric, anti-symmetric, transitive. Explain each of your verdicts.

   **Comment**: be ready to do item d) using any of the three representations you received in items a) - b).

3. Relations $R$ and $S$ on set $A = \{a, b, c\}$ are represented by the matrices.

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \texttt{and} \quad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

   **a)** Give digraphs representing relations $R$ and $S$

   **b)** Find $R \circ S$ using matrices

   **c)** Find $R \circ S$ using digraphs

   **d)** Compare the results you got in **b)** and **c)**

   **e)** Find $S \circ R$ using matrices

   **f)** Find $S \circ R$ using digraphs

   **g)** Compare the results you got in **d)** and **e)**

4. The relation $S$ on set $A = \{x, y, z\}$ is represented by the matrix.

$$M_S = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

**a)** Give the *arrow diagram* of the relation (digraph)

**b)** Determine whether the relation $S$ is reflexive, anti-reflexive, symmetric, anti-symmetric, transitive. Explain your answer.

**c)** Find $M_S^2$, then draw the ***arrow diagram*** of the composition $S \circ S$.

**d)** Find $S^+$, the ***transitive closure of the relation*** $S$.

**Comments:**

1) review the transitive closure for directed graphs with finite number of vertices

2) be ready to find the transitive closure of digraphs and relations.

3) review the Graph Power Theorem

5. Is $(\mathbf{Z},<)$ a ***poset***? Is it a ***total order***? Explain.

6. Is $(\mathbf{Z},<)$ a ***strict order***? Is it a ***total order***? Explain.

7. Consider the following relation:

   the domain is the set of all positive integers.

   $x$ is related to $y$ if there exist some positive integer $a$ such that $y = 5 \cdot a \cdot x$.

   Is this relation a ***partial order***, a ***strict order***, or ***neither***?

   If the relation is a ***partial*** or ***strict order***, is it also a ***total order***?

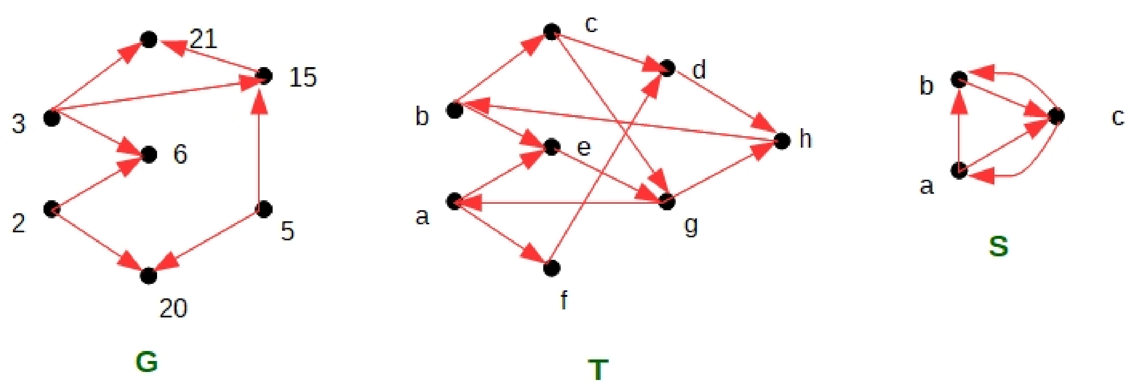
   Explain.

8. Draw the *Hasse diagram* for divisibility on the set $\{1, 2, 3, 4, 12, 24, 36, 48\}$, then

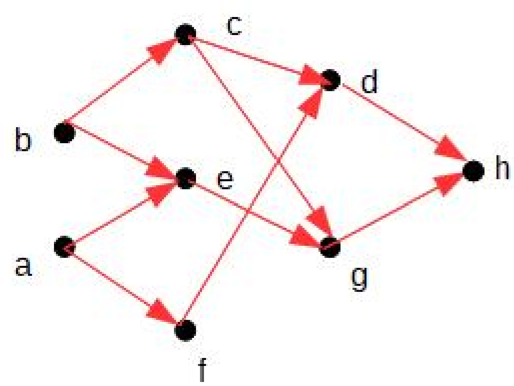
   **a)** Find the maximal elements

   **b)** Find the minimal elements

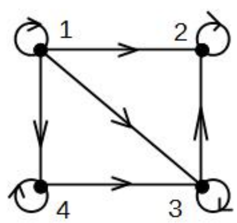
9. Which of the given graphs are DAGs (Directed Acyclic Graph)? Explain.



10. Give a topological sort of the directed acyclic graph shown below




11. Consider the relation represented by the directed graph




**a)** Determine whether the relation is an equivalence relation. Explain.

**b)** Determine whether the relation is a partial order relation. Explain.

12. Relation $R$ consisting of all pairs $(x, y)$ such that $x$ and $y$ are bit strings of length seven that agree in their last four bits. Is it an equivalence relation of the set of all bit strings of length seven? Explain your answer.

13. List the ordered pairs in the equivalence relation produced by the partition $\{a\}, \{b, c, d\}, \{e, f\}$ of the set $\{a, b, c, d, e, f\}$

14. The domain of the equivalence relation $R$ is the set S:

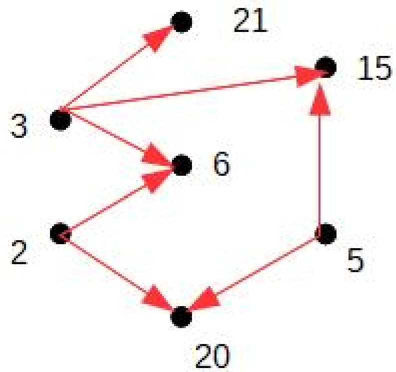
$S = \{1, 2, 3, 8, 10, 15, 23, 30, 35, 45\}$

For any $x, y \in S$, $xRy$ if $x \equiv y (\texttt{mod } 7)$.

Show the partition of $S$ defined by the equivalence classes of $R$.

# 2 Answers

1.

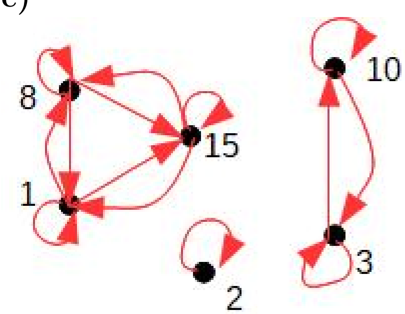



2. **a)** $R = \{(1,8),(8,1),(8,15),(15,8),(1,15),(15,1),(3,10),(10,3),(1,1),(2,2),(3,3),(8,8),(10,10),(15,15)\}$

**b)**

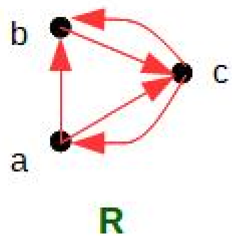
$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

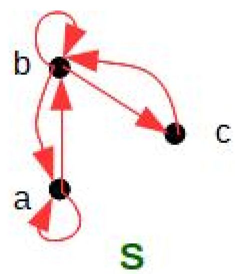
**c)**




**d)** The relation $R$ is reflexive, symmetric, not anti-symmetric, and transitive.

3. **a)**




**b)** $M_{R \circ S} = M_S \times M_R =$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

**c)** $R \circ S = \{(a, b), (a, c), (b, a), (b, b), (b, c), (c, c)\}$

**d)** the results in **b)** and **c)** are equivalent

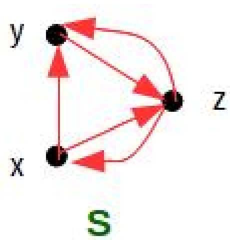
**e)** $M_{S \circ R} = M_R \times M_S =$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

**f)** $S \circ R = \{(a, a), (a, b), (a, c), (b, b), (c, a), (c, b), (c, c)\}$

**g)** the results in **e)** and **f)** are equivalent

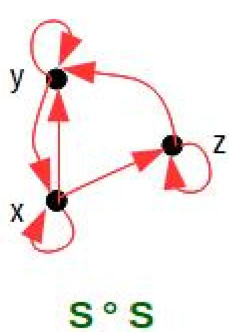
4. **a)**




**b)** The relation $S$ is not reflexive, anti-reflexive, not symmetric, not anti-symmetric, and not transitive.

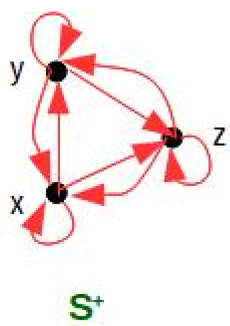
**c)** $M_S^2 =$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$



**d)** $M_{S^+} =$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$




5. The $(Z, <)$ is not a poset (not reflexive)

6. The $(Z, <)$ is a strict order and a total order

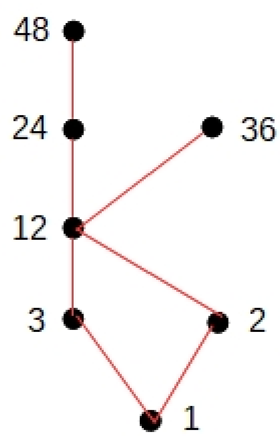
7. The given relation is a strict order, but not a total order (2 and 3 are incomparable)

   anti-reflexive (because $(x, x) \notin R$, i.e. $x! = 5 \cdot a \cdot x$, where $a > 0$)

   anti-symmetric (because $(2, 20) \in R$, but $(20, 2) \notin R$, i.e. $20 = 5 \cdot 2 \cdot 2$ but $2 = 5 \cdot ? \cdot 20$)

   transitive (because if $(x, y) \in R$ and $(y, z) \in R$, then $y = 5 \cdot a_1 \cdot x$ and $z = 5 \cdot a_2 \cdot y$, hence $z = 5 \cdot a_2 \cdot 5 \cdot a_1 \cdot x$, therefore $(x, z) \in R$)

8. Hasse diagram:




   maximal elements: $48, 36$

   minimal element: $1$

9. Only graph $G$ is a DAG.

10. There are many topological sorts, here are two of them:

    $\{a, b, c, e, f, d, g, h\}$

    $\{a, f, b, c, e, d, g, h\}$

11. **a)** the given relation is not an equivalence relation because it is not symmetric: $(1,4)$ is present, but $(4,1)$ is not

    **b)** the given relation is not a partial order, because it is not transitive.

    - reflexive: $(1,1),(2,2),(3,3),(4,4) \in R$
    - anti-symmetric: no pairs $(a,b)$ and $(b,a)$ where $a \neq b$ are present
    - not transitive: $(4,3),(3,2) \rightarrow (4,3)$, which is not present

12. relation R is an equivalence relation

    - reflexive: $(x,x) \in R$, because the same strings do agree on the last four bits
    - symmetric: if $(x,y) \in R$, the obviously $(y,x) \in R$
    - transitive: if two bit-strings, $x$ and $y$ agree on their last four bits, and strings $y$ and $z$ agree on their last four bits, then obviously $x$ and $z$ should agree on their last four bits.

13. $R = \{(a,a),(b,b),(b,c),(b,d),(c,b),(c,c),(c,d),(d,b),(d,c),(d,d),(e,e),(e,f),(f,e),(f,f)\}$

14. the partitions are: $\{1,8,15\}$, $\{2,23,30\}$, $\{3,10,45\}$, $\{35\}$