BRONX COMMUNITY COLLEGE

of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

CSI 35 Test 1 Review Questions

1 Chapter 5: Induction and Recursion

1. Compute
$$\sum_{i=5}^{9}(-1)^i\frac{(i-7)^2}{10}$$
Write your answer in simplified fractional form.

2. Compute
$$\sum_{i=12}^{i=128} 17$$

3. Compute
$$\sum_{i=-2}^{26}(5i-8)$$

4. Rewrite
$$\sum_{j=-3}^{n+1}(j+5)^3$$
so that the lower limit is 1

5. Find the sum $15+19+23+27+\ldots+403$

6. Find the sum $3+(-12)+48+\ldots+12,288$

The next three problems are proofs by *mathematical induction.* Follow the sketch:

1) Identify the statement $P(n)$:

2) **Basis step:**

3) **Inductive step:** state the IH (Inductive Hypotheses first)

4) Show what needs to be proved, i.e. state $P(k+1)$:

5) present the proof (don't forget to show where the IH is used):

6) State the conclusion:

7) Put **q.e.d.** at the end.

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7. Use *mathematical induction* to show that

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

   whenever n is a positive integer.

8. Use *mathematical induction* to show that $2^n > n^2+n$ for all integer values $n > 4$

9. Use *mathematical induction* to prove that 9 divides $n^3+(n+1)^3+(n+2)^3$ for all non-negative integers n.

10. If the following proof correct? Justify your answer.

    Let's prove that $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(n-1)n}=\frac{3}{2}-\frac{1}{n}$, if $n \in Z^+$

    Let $P(n)$ stand for $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(n-1)n}=\frac{3}{2}-\frac{1}{n}$

    Basis step: The result is true when $n=1$ because $\frac{1}{1\cdot 2}=\frac{3}{2}-\frac{1}{1}$.

    Inductive step: assume that the result, $P(k)$, is true for an integer value $k \geq 1$ (**IH**), i.e.

    $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(k-1)k}=\frac{3}{2}-\frac{1}{k}$.

    Let's show that in this case $P(k+1)$ is also true:

    $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(k-1)k}+\frac{1}{((k+1)-1)(k+1)}=\text{by } \mathbf{IH} = \frac{3}{2}-\frac{1}{k}+\frac{1}{((k+1)-1)(k+1)}=$
    $\frac{3}{2}-\frac{1}{k}+\frac{1}{k(k+1)}=\frac{3}{2}+\frac{-(k+1)+1}{k(k+1)}=\frac{3}{2}+\frac{-k}{k(k+1)}=\frac{3}{2}-\frac{1}{k+1}$

    Therefore, we proved that $P(k+1)$ is also true, i.e.

    $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(k-1)k}+\frac{1}{((k+1)-1)(k+1)}=\frac{3}{2}-\frac{1}{k+1}$

    This completes the inductive step.

    By mathematical induction we proved that

    $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(n-1)n}=\frac{3}{2}-\frac{1}{n}$, if $n \in Z^+$

    q.e.d.

11. Determine whether the proposed definition is a valid recursive definition of a function $f: Z^+ \to Z$. If f is well defined, find a non-recursive definition for $f(n)$ when $n \in Z^+$.

    $f(1)=1$,

    $f(2)=0$,

    $f(3)=2$,

    $f(n)=2f(n-3)$, for $n \geq 4$

12. Give a recursive definition of $P_m(n)$, the product of the integer m and the non-negative integer n.

13. Give a recursive definition of the set S^* of all binary strings of positive length starting with digit 1.

    for example: $1 \in S^*$, $101 \in S^*$, $101010111 \in S^*$, but $0101 \notin S^*$

14. Given a recursive algorithm below, trace the call of ***thing(4)***, i.e. show all the steps used by the algorithm.

    Input: a positive integer n

    procedure *thing*(n):

    if $n == 1$ **then return** 1

    else return $1 + 2 * thing(n-1)$

15. Write a ***recursive*** algorithm calculating the sum $\sum_{i=1}^{n}(3i+5)^2$, where n is a positive integer.