CS135 Midterm Exam Sample

N4. $9 \mid n^3 + (n+1)^3 + (n+2)^3 \leftarrow P(n)$, for all $n \in \mathbb{Z}$, $n \geq 0$

<u>Proof:</u> <u>Basis step:</u> let's show that $P(0)$ is true:

$0^3 + (0+1)^3 + (0+2)^3 = 1 + 8 = 9 \qquad 9 \mid 9$

<u>Inductive step:</u> assume that for any arbitrary fixed $k \geq 0$, $P(k)$ is true, i.e. $9 \mid k^3 + (k+1)^3 + (k+2)^3$. (IH)

Let's show that in this case $P(k+1)$: $9 \mid (k+1)^3 + (k+2)^3 + (k+3)^3$ is also true:

$$(k+1)^3 + (k+2)^3 + (k+3)^3 = (k+1)^3 + (k+2)^3 + k^3 + 3 \cdot k^2 \cdot 3 + 3 \cdot k \cdot 3^2 + 3^3 =$$

$$= k^3 + (k+1)^3 + (k+2)^3 + 9k^2 + 27k + 27 =$$

$$= \underbrace{\underbrace{k^3 + (k+1)^3 + (k+2)^3}_{\text{divisible by 9 by (IH)}} + \underbrace{9(k^2 + 3k + 3)}_{\text{divisible by 9}}}_{\text{divisible by 9 by theorem}}$$

Therefore, we ~~demonstrated~~ showed that $9 \mid (k+1)^3 + (k+2)^3 + (k+3)^3$, i.e. $P(k+1)$ is true

This completes the inductive step.

By mathematical induction we proved that $9 \mid n^3 + (n+1)^3 + (n+2)^3$ for all non-negative integers n.

q.e.d.