CS135 Midterm Exam v2

N3 $\dfrac{1}{1\cdot 3} + \dfrac{1}{3\cdot 5} + \dots + \dfrac{1}{(2n-1)(2n+1)} = \dfrac{n}{2n+1}$ $P(n)$

$n \in \mathbb{Z}^+$

Proof:

Basis step: let's check $P(1)$: $\dfrac{1}{1\cdot 3} \stackrel{?}{=} \dfrac{1}{2\cdot 1+1}$ Yes

therefore $P(1)$ is true

Inductive step: assume that for any arbitrary fixed $k \in \mathbb{Z}^+$

$P(k)$ is true, i.e. $\dfrac{1}{1\cdot 3} + \dfrac{1}{3\cdot 5} + \dots + \dfrac{1}{(2k-1)(2k+1)} = \dfrac{k}{2k+1}$ (IH)

Let's show that in this case $P(k+1)$ is also true, i.e.

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)} = \frac{k+1}{2(k+1)+1}$$

Let's start with the left side of the equality:

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)} =$$

$$= \underbrace{\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \dots + \frac{1}{(2k-1)(2k+1)}} + \frac{1}{(2k+1)(2k+3)} =$$

$$= \underbrace{\frac{k}{2k+1}}_{\text{by (IH)}} + \frac{1}{(2k+1)(2k+3)} = \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)} =$$

$$= \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \frac{k+1}{2k+3} = \frac{k+1}{2(k+1)+1}$$ – we showed that $P(k+1)$ is true

This concludes inductive ~~prove~~ step.

By mathematical induction, $P(n)$ is true for all $n \in \mathbb{Z}^+$.

q.e.d.