

Let's prove that $2^n > n^2 + 4$ for all integer values $n > 4$.

1. Base step: checking for $n=5$:

$2^5 = 32$ and $5^2+4 = 29$, and $32 > 29$ is true

2. Inductive step:

Let's assume that for an arbitrary integer $k > 4$, $2^k > k^2 + 4$ is true. (this is our *Inductive Hypotheses*)

Let's show that under our inductive hypotheses, $2^{k+1} > (k+1)^2 + 4$ is also true:

$2^{k+1} = 2 \cdot 2^k > (\text{by IH}) 2 \cdot (k^2+4) = 2k^2 + 8 > k^2 + 2k + 8 > k^2 + 2k + 5 = (k+1)^2 + 4$

Note that $(k+1)^2 + 4 = k^2 + 2k + 5$, and $k^2 > 2k$ for integer values greater than 2

$3^2 > 2 \cdot 3$, $4^2 > 2 \cdot 4$, etc

We showed that under the IH, $2^{k+1} > (k+1)^2 + 4$.

This completes the inductive step.

By mathematical induction we proved that $2^n > n^2 + 4$ for all integer values $n > 4$.