BRONX COMMUNITY COLLEGE

of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

CSI 35 Midterm Exam Sample

Present all your work in order to get full credit.
Show solutions, give explanations. Don't give just answers.

1. Compute
$$\sum_{i=12}^{i=128} 17$$

2. Compute
$$\sum_{i=5}^{9} (-1)^i \frac{(i-7)^2}{10}$$
Write your answer in simplified fractional form.

3. Find the sum $3 + (-12) + 48 + \ldots + 12,288$

For the next problem, a proof by *mathematical induction*, follow the sketch:

1) Identify the statement $P(n)$:

2) **Basis step:**

3) **Inductive step:** state the IH (Inductive Hypotheses first)

4) Show what needs to be proved, i.e. state $P(k+1)$:

5) present the proof (don't forget to show where the IH is used):

6) State the conclusion:

7) Put **q.e.d.** at the end.

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4. Use *mathematical induction* to prove that 9 divides $n^3 + (n+1)^3 + (n+2)^3$ for all non-negative integers n.

5. Determine whether the proposed definition is a valid recursive definition of a function $f : Z^+ \to Z$. If f is well defined, find a formula for $f(n)$ when $n \in Z^+$.
$f(1) = 1$,
$f(2) = 0$,
$f(3) = 2$,
$f(n) = 2f(n-3)$, for $n \geq 4$

6. Give a recursive definition of $P_m(n)$, the product of the integer m and the non-negative integer n.

7. Given a recursive algorithm below, trace the call of ***thing(4)***, i.e. show all the steps used by the algorithm.

   Input: a positive integer n

   procedure *thing*(n):

   if $n == 1$ **then return** 1

   else return $1 + 2{*}thing(n-1)$

8. For the relation $R = \{(a,b)|a \equiv b(\texttt{mod } 7)\}$ on the set $\{1,2,3,8,10,15\}$

   a) List all the ordered pairs in R

   b) Give matrix representation of relation R

   c) Give digraph representing relation R

   d) Determine whether relation R is reflexive, symmetric, antisymmetric, asymmetric, and/or transitive. Explain each of your verdicts.

9. The relation S on set A is represented by the matrix.

$$M_S = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

   a) Find S^{-1}

   b) Find $\overline{S}$

10. Relations R and S on set A are represented by the matrices.

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \texttt{and} \quad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

   a) Find $R \cup S$

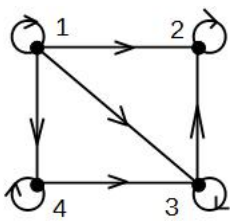
   b) Find $R \cap S$

   c) Find $S \circ R$

   d) Find $S - R$

11. Relation R consisting of all pairs (x, y) such that x and y are bit strings of length seven that agree in their last four bits. Is it an equivalence relation of the set of all bit strings of length seven? Explain your answer.

12. Consider the relation represented by the directed graph



   a) Determine whether the relation is an equivalence relation. Explain.

   b) Determine whether the relation is a partial order relation. Explain.

13. List the ordered pairs in the equivalence relation produced by the partition $\{a\}, \{b, c, d\}, \{e, f\}$ of the set $\{a, b, c, d, e, f\}$

14. Is $(\mathbf{Z}, <)$ a poset? Explain.

15. Draw the *Hasse diagram* for divisibility on the set $\{1, 2, 3, 4, 12, 24, 36, 48\}$, then

    a) Find the maximal elements

    b) Find the minimal elements

    c) Is there a greatest element?

    d) Is there a least element?

    e) Find all upper bounds of $\{2, 3\}$

    f) Find all lower bounds of $\{12, 24\}$