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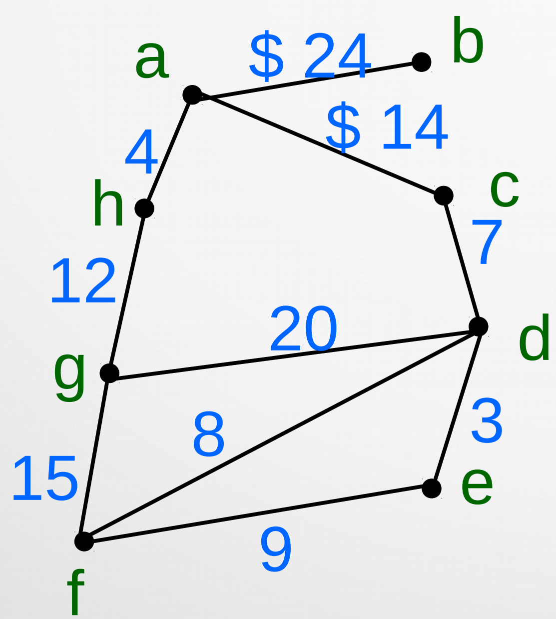

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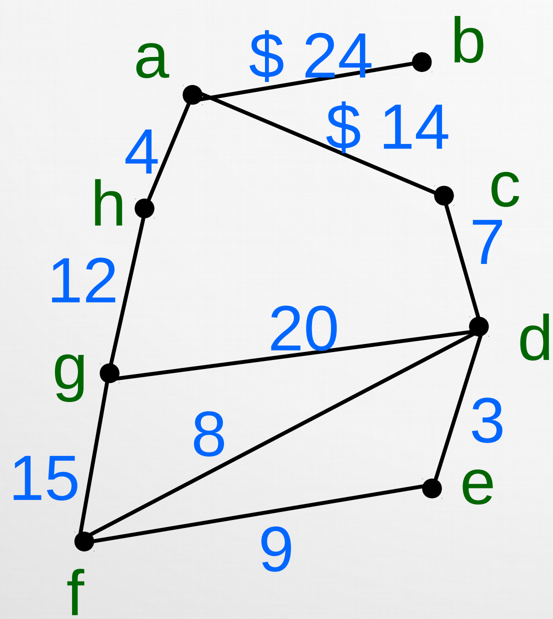

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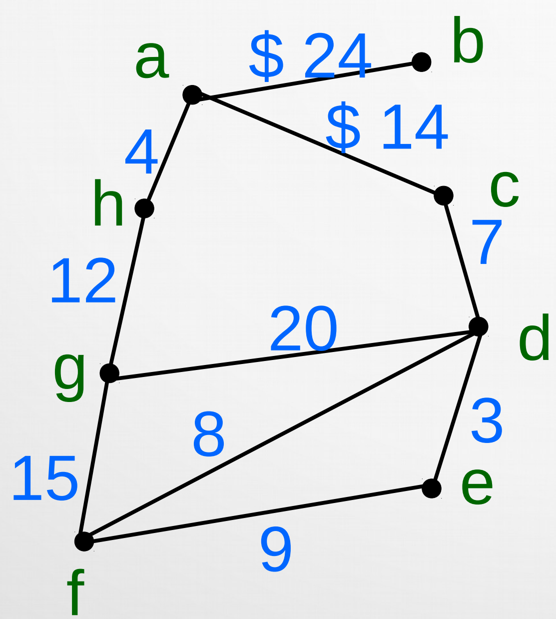

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procedure *Prim*(*G*: weighted connected undirected graph with *n* vertices)
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We will assume that the edges are ordered when we need to choose between two or more edges with the same weights.

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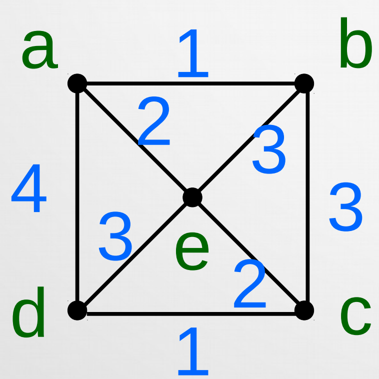

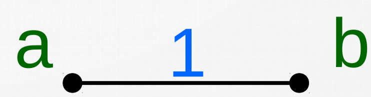

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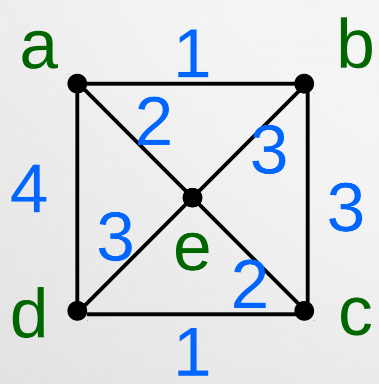

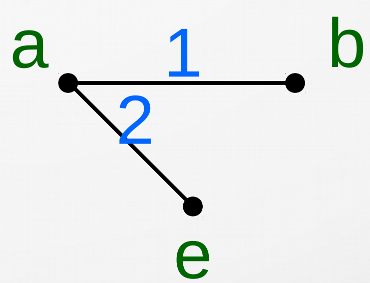

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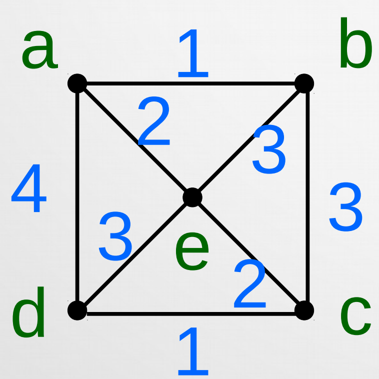

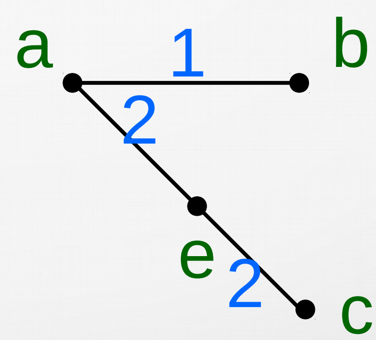

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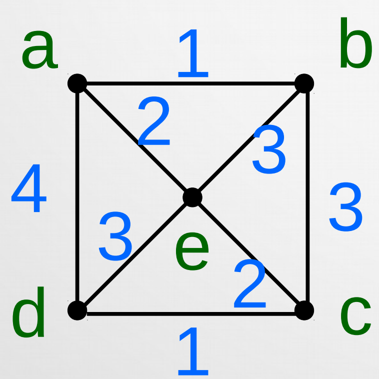

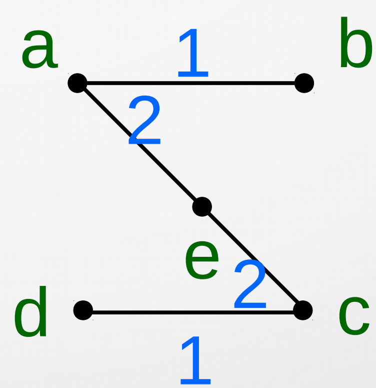

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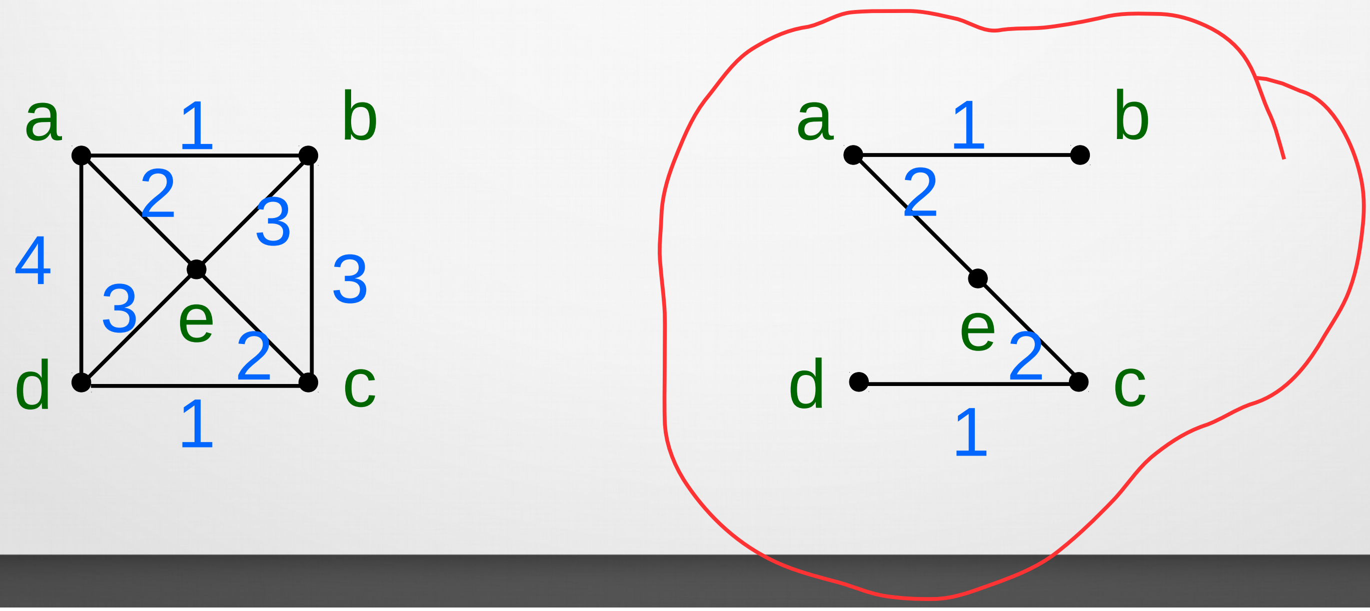

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Historical information:

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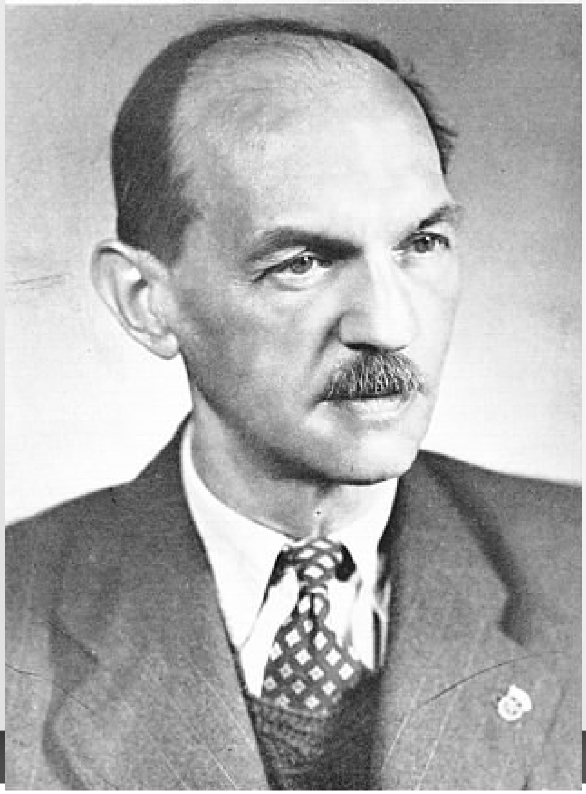

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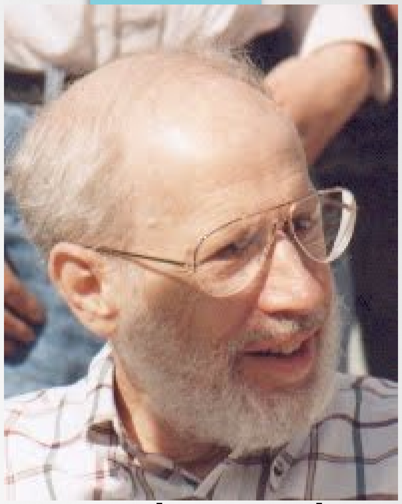
During his career at Bell Laboratories, Robert Prim along with coworker Joseph Kruskal developed two different algorithms for finding a minimum spanning tree in a weighted graph

11.5 *Minimum Spanning Trees*

Kruskal's algorithm



The second algorithm up for discussion was discovered by Joseph Kruskal in 1956, although the basic ideas it uses were described much earlier.

procedure *Kruskal*(*G*: weighted connected undirected graph
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for *i* := *1* to *n-1*
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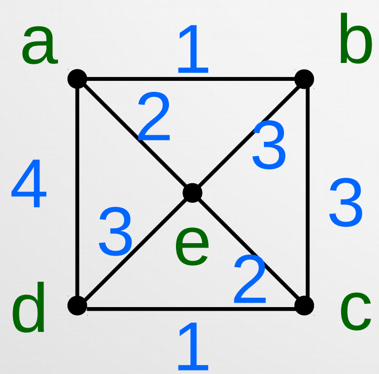

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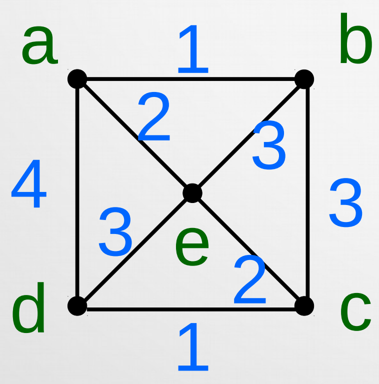

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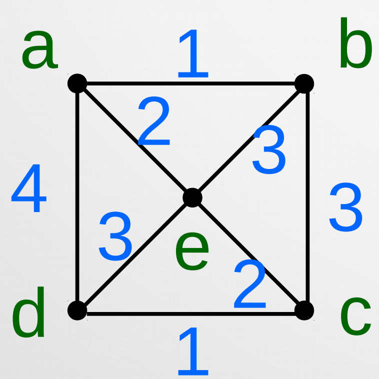

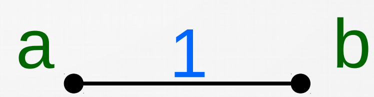

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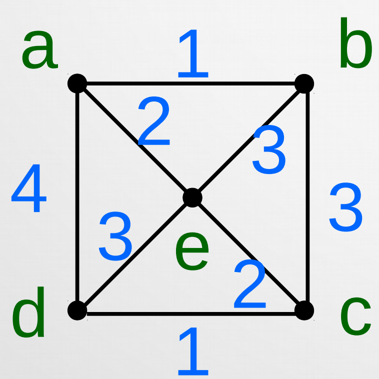

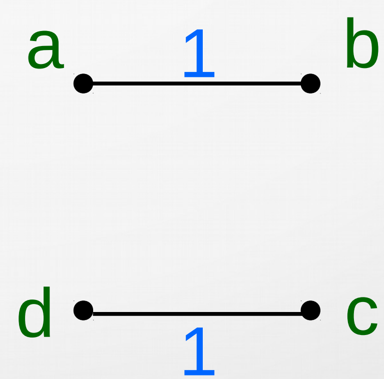

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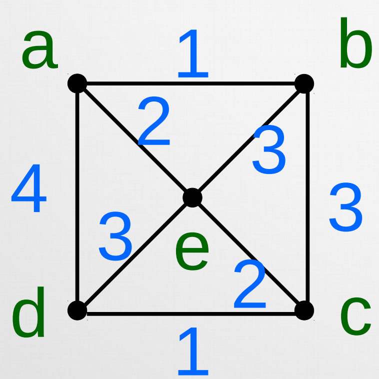

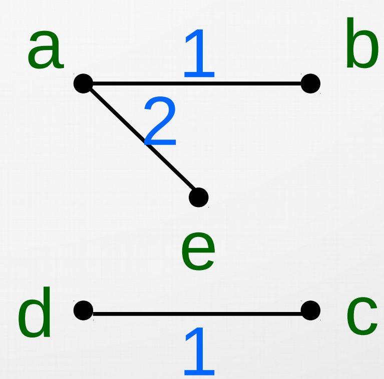

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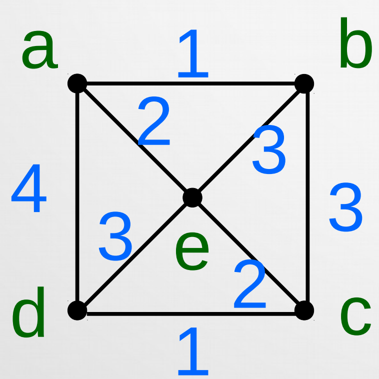

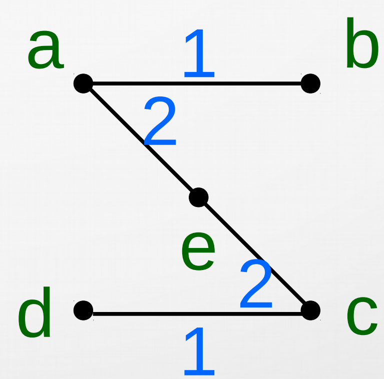

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