Today we will discuss section:

**Section 10.8** *Graph coloring*

but we will grab a definition from **Section 10.7**

# 10.8 *Graph coloring*

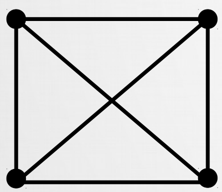
**[Def]** a graph is called *planar* if it can be drawn in the plane without edges crossing.

Such a drawing is called a *planar representation of the graph*.

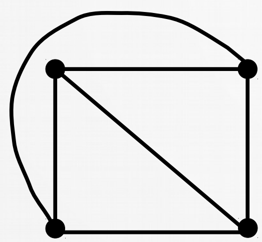
# 10.8 *Graph coloring*

**[Def]** a graph is called *planar* if it can be drawn in the plane without edges crossing.

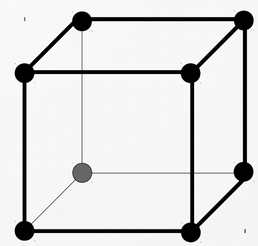
Such a drawing is called a *planar representation of the graph*.



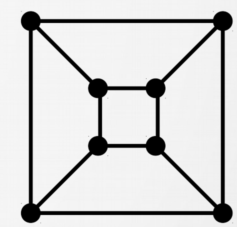
$K_4$



a planar representation of $K_4$



$Q_3$



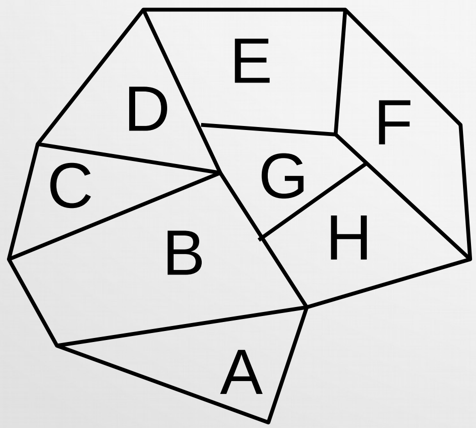
a planar representation of $Q_3$

# 10.8 *Graph coloring*

## Map coloring

Assume that we want to color a map with a smallest number of colors used so that no two adjacent regions have the same color.

*exception*: two regions that touch only at one point are not considered adjacent.

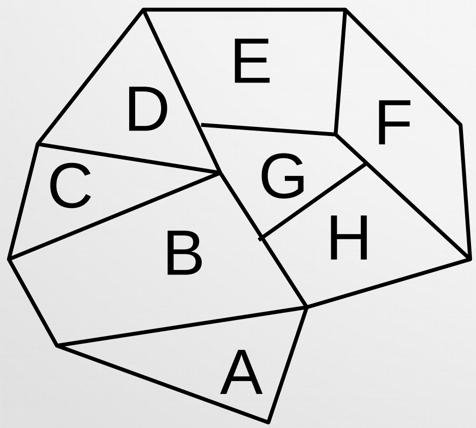


# 10.8 *Graph coloring*

## Map coloring

Assume that we want to color a map with a smallest number of colors used so that no two adjacent regions have the same color.

*exception*: two regions that touch only at one point are not considered adjacent.




Each map on the plane can be represented by a graph.

vertices: regions
edges: common borders

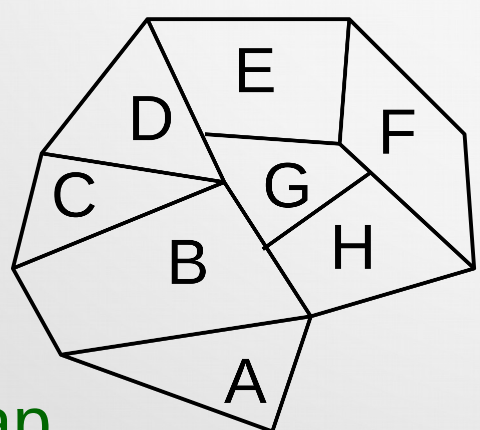
*regions that touch only at one point are not considered adjacent*

# 10.8 *Graph coloring*

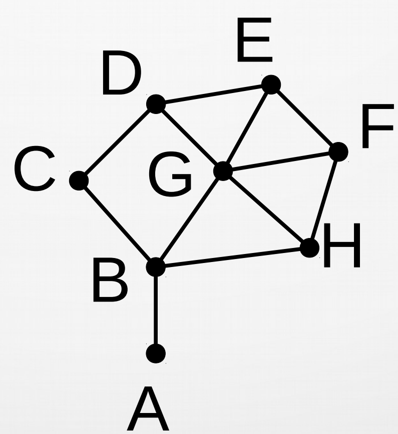
## Map coloring

Assume that we want to color a map with a smallest number of colors used so that no two adjacent regions have the same color.

*exception*: two regions that touch only at one point are not considered adjacent.




map



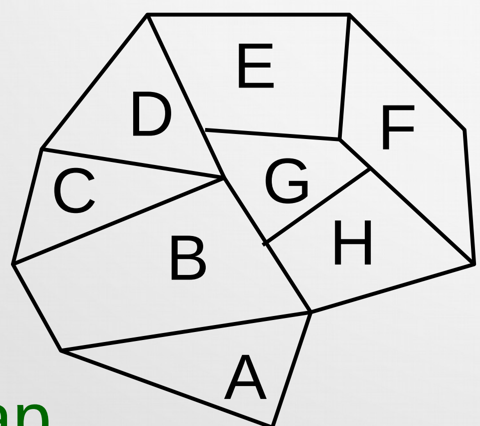

*dual graph* of the map

# 10.8 *Graph coloring*

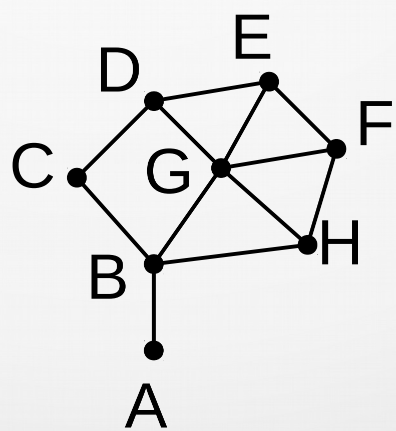
## Map coloring

Assume that we want to color a map with a smallest number of colors used so that no two adjacent regions have the same color.

*exception*: two regions that touch only at one point are not considered adjacent.




map




*dual graph* of the map

any map in the plane has a planar dual graph

# 10.8 *Graph coloring*

## Map coloring

Assume that we want to color a map with a smallest number of colors used so that no two adjacent regions have the same color.

*exception*: two regions that touch only at one point are not considered adjacent.

The problem of coloring the regions of a map is *equivalent* to the problem of coloring the vertices of the dual graph so that no adjacent vertices in this graph have the same color.

# 10.8 *Graph coloring*

## Map coloring

**[Def]** A *coloring of a simple graph* is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

# 10.8 *Graph coloring*

## Map coloring

**[Def]** A *coloring of a simple graph* is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

**[Def]** A *chromatic number of a graph* is the least number of colors needed for coloring of this graph.

denotaion: $\chi(G)$

# 10.8 *Graph coloring*

## Map coloring

**[Def]** A *coloring of a simple graph* is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

**[Def]** A *chromatic number of a graph* is the least number of colors needed for coloring of this graph.

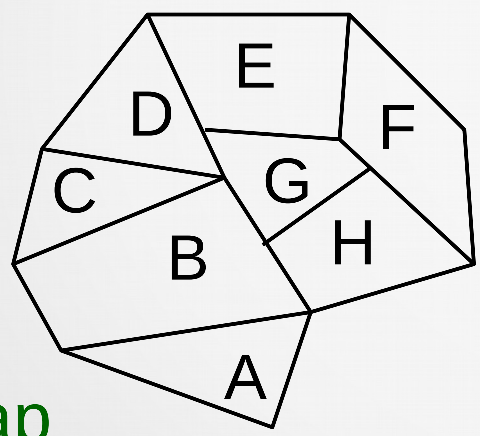
denotaion: $\chi(G)$

**[Theorem]** The Four Color Theorem
The chromatic number of a planar graph is no greater than four.

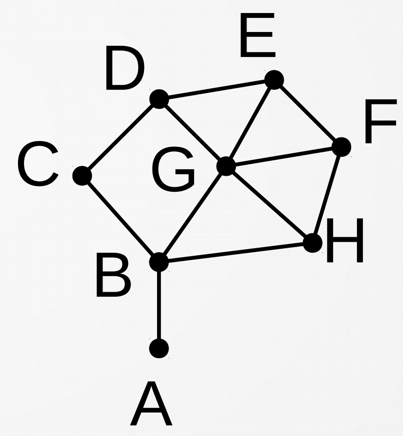
# 10.8 *Graph coloring*

## Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map



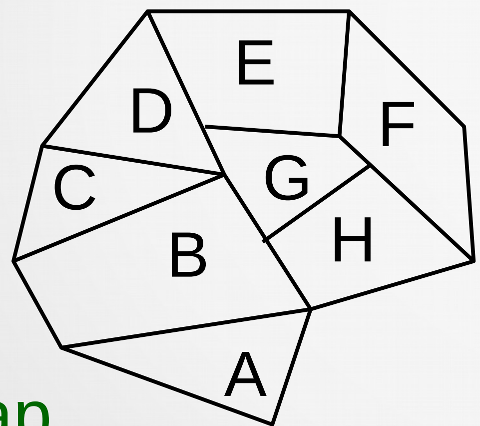

*dual graph* of the map G

Two things to do:
1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

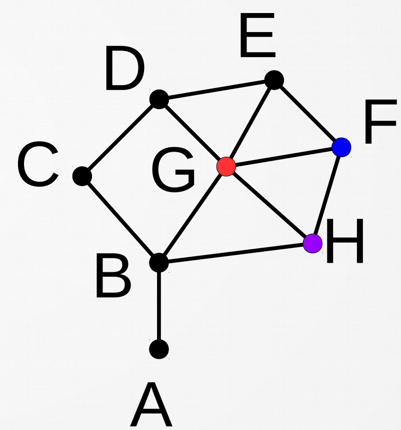
# 10.8 *Graph coloring*

## Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map




*dual graph* of the map G

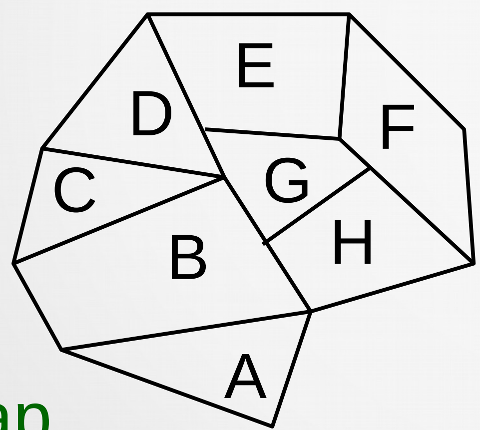
Two things to do:

1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

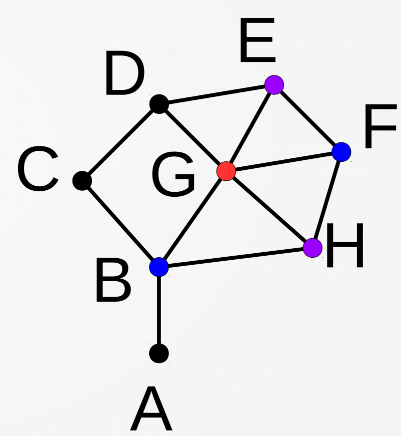
# 10.8 *Graph coloring*

Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map



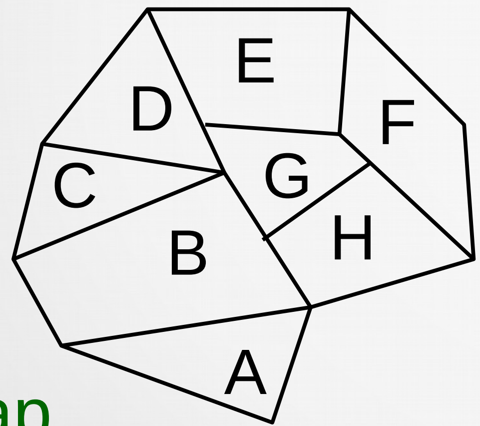

*dual graph* of the map G

Two things to do:
1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

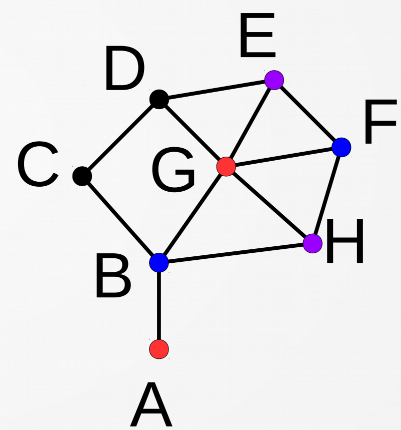
# 10.8 *Graph coloring*

## Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map




*dual graph* of the map G

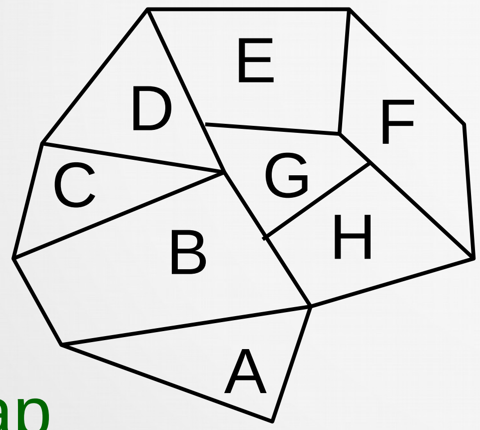
<u>Two things to do</u>:

1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

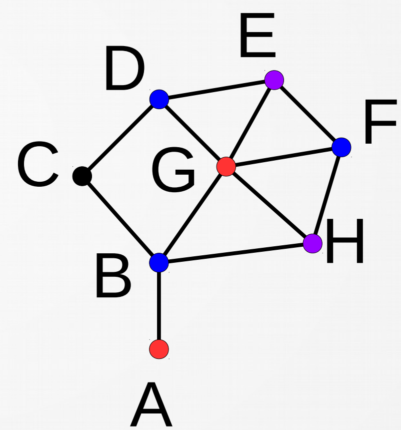
# 10.8 *Graph coloring*

Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map



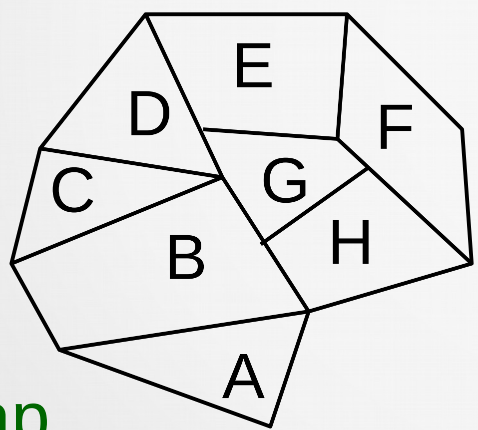

*dual graph* of the map
G

Two things to do:
1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

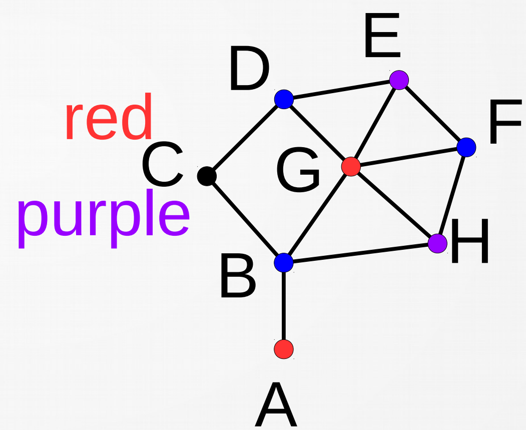
# 10.8 *Graph coloring*

## Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map



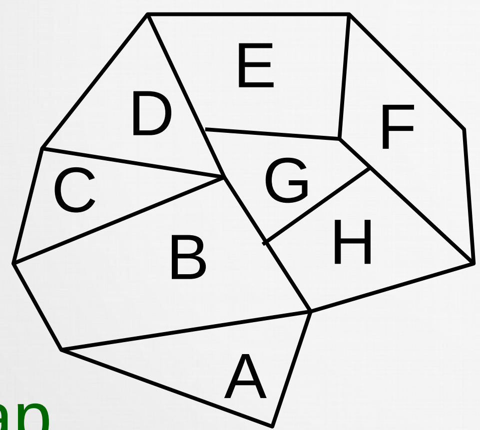

*dual graph* of the map
G

Two things to do:
1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

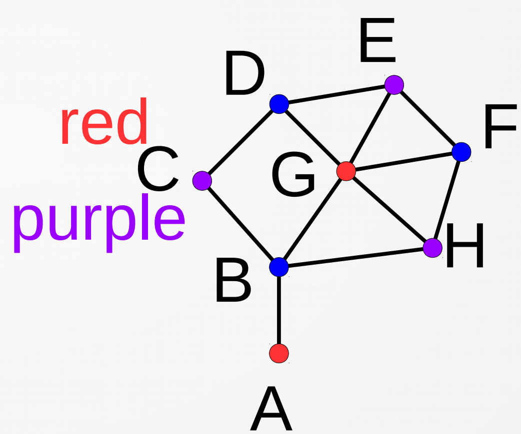
# 10.8 *Graph coloring*

## Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map



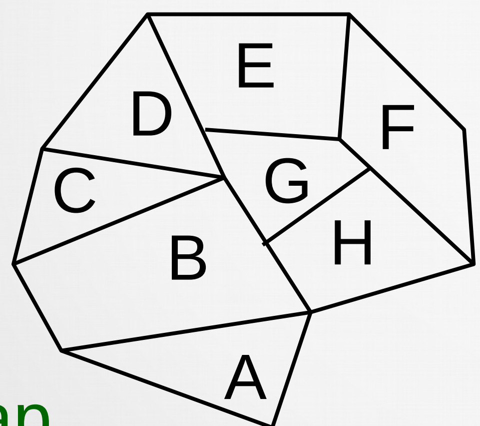

*dual graph* of the map G

Two things to do:
1) show that a graph G can be colored with *k* colors
2) show that G cannot be colored with fewer than *k* colors

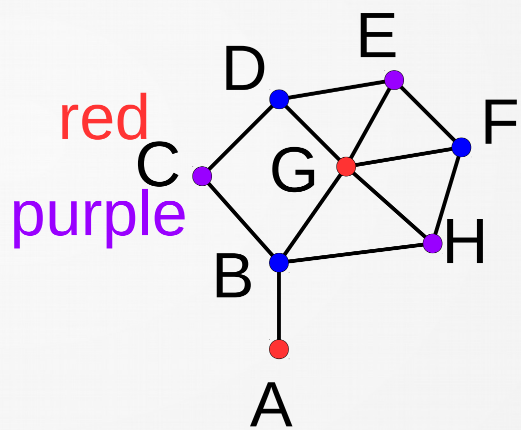
# 10.8 *Graph coloring*

## Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map




*dual graph* of the map G

minimum: 3 colors

<u>Two things to do</u>:

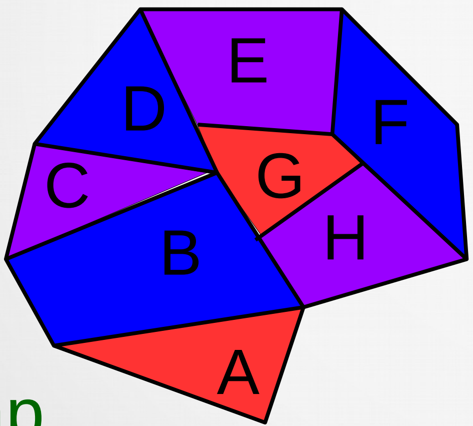
1) show that a graph G can be colored with *k* colors

2) show that G cannot be colored with fewer than *k* colors

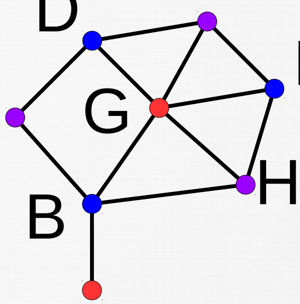
# 10.8 *Graph coloring*

Map coloring

**Example**: find the chromatic number for graph G, then color the map so that no two adjacent regions have the same color.




map




*dual graph* of the map G

minimum: 3 colors

<u>Two things to do</u>:

1) show that a graph G can be colored with *k* colors

2) show that G cannot be colored with fewer than *k* colors

# 10.8 *Graph coloring*

## Map coloring

**[Theorem]** The Four Color Theorem
The chromatic number of a planar graph is no greater than four.

The theorem was originally posted as a conjecture in the 1850s, and was finally proved by American mathematicians Kenneth Appel and Wolfgang Haken in 1976.

Read about it on the page 728.

# 10.8 *Graph coloring*

## Map coloring

The best algorithms known for finding the chromatic number of a graph have exponential worst-case time complexity (in the number of vertices of the graph).

Even the problem of finding an approximation to the chromatic number of a graph is difficult.

## Applications of graph coloring

**Scheduling Final Exams**

How can the final exams at a university be scheduled so that no student has two exams at the same time?

vertices: courses
edges: if there is a student taking both courses

Each time slot for a final exam is represented by a different color.

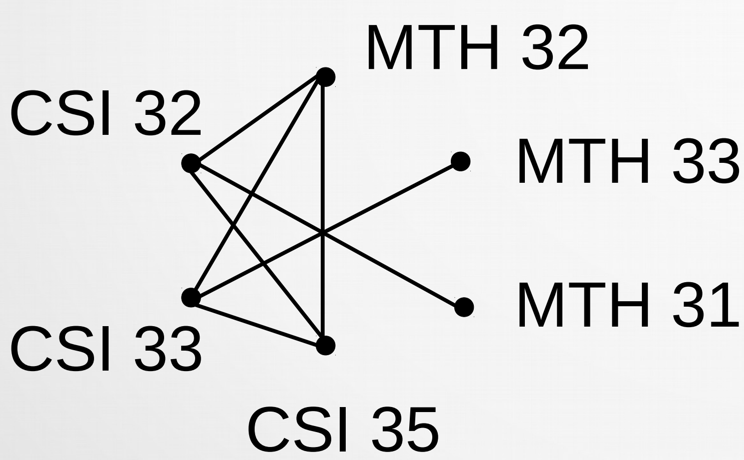
Then scheduling corresponds to a coloring of the associated graph.

# 10.8 *Graph coloring*

## Applications of graph coloring

**Scheduling Final Exams**

Assume the following is true:



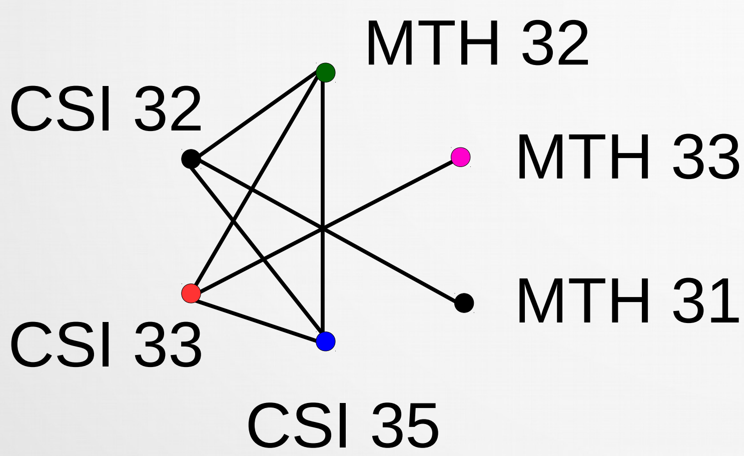

Time period:

# 10.8 *Graph coloring*

## Applications of graph coloring

**Scheduling Final Exams**

Assume the following is true:



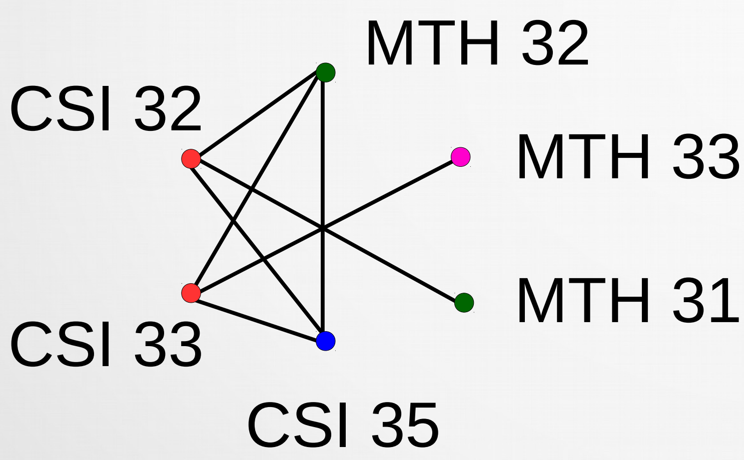

Time period:

# 10.8 *Graph coloring*

## Applications of graph coloring

**Scheduling Final Exams**

Assume the following is true:




Time period:

| | |
|---|---|
| I | CSI33, CSI32 |
| II | MTH32, MTH31 |
| III | CSI 35 |
| IV | MTH 33 |

# 10.8 *Graph coloring*

## Applications of graph coloring

**Frequency Assignment**

Television channels 2 through 13 are assigned stations in North America so that no two stations within 150 miles can operate on the same channel.
How can the assignment of channels be modeled by graph coloring?

# 10.8 *Graph coloring*

## Applications of graph coloring

**Frequency Assignment**

Television channels 2 through 13 are assigned stations in North America so that no two stations within 150 miles can operate on the same channel.
How can the assignment of channels be modeled by graph coloring?

Answer:
vertices: stations
edges: between two stations within 150 miles of each other

An assignment: each color represents a different channel

# 10.8 *Graph coloring*

## Applications of graph coloring

**Index Registers**

In efficient compilers the execution of loops is speeded up when frequently used variables are stored temporarily in index registers in CPU, instead of in regular memory. For a given loop, how many index variables are needed?

# 10.8 *Graph coloring*

## Applications of graph coloring

**Index Registers**

In efficient compilers the execution of loops is speeded up when frequently used variables are stored temporarily in index registers in CPU, instead of in regular memory. For a given loop, how many index variables are needed?

Answer: can be modeled by graph coloring
vertices: a variable in the loop
edges: if two variables need to be stored in index registers at the same time during the execution of the loop.
Chromatic number gives the # of index registers needed.