Section 9.5 *Practice*

Example 1: Consider the relation

$$R = \{(f,g) \mid \exists C \in \mathbf{Z}, \forall x \in \mathbf{Z}, f(x) - g(x) = C\}$$

on the set of all functions from **Z** to **Z**.

Is it an equivalence relation?

If it is not, what properties is it lacking?

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If $(f,g) \in R$ then $\exists C \in \mathbf{Z}, \forall x \in \mathbf{Z}, f(x) - g(x) = C$

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If $(f,g) \in R$ then $\exists C \in \mathbf{Z}, \forall x \in \mathbf{Z}, f(x) - g(x) = C$
In this case $g(x) - f(x) = -C$, and $-C \in \mathbf{Z}$, so $(g,f) \in R$

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In this case $g(x) - f(x) = -C$, and $-C \in \mathbf{Z}$, so $(g,f) \in R$
Therefore, yes it is *symmetric*.

Section 9.5 *Practice*

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$$R = \{(f,g) \mid \exists C \in \mathbf{Z}, \forall x \in \mathbf{Z}, f(x) - g(x) = C\}$$

on the set of all functions from **Z** to **Z**.

Is it an equivalence relation?
If it is not, what properties is it lacking?

Is it *transitive*, i.e. if $(f,g) \in R$ and $(g,m) \in R$, does $(f,m) \in R$?

$(f,g) \in R$: $\exists C_1 \in \mathbf{Z}, \forall x \in \mathbf{Z}, f(x) - g(x) = C_1$

$(g,m) \in R$: $\exists C_2 \in \mathbf{Z}, \forall x \in \mathbf{Z}, g(x) - m(x) = C_2$

Hence $f(x) - m(x) = C_1 + C_2$, and $C_1 + C_2 \in \mathbf{Z}$, thus $(f,m) \in R$

Therefore, yes it is *transitive*.

Section 9.5 *Practice*

Example 2: bit-strings

Bit-strings are composed of 0s and 1s.

10101

11101

101

…

The length of a bit-string is the number of digits in it.

Let $n \in \mathbf{Z}^+$ and S be a set of all bit-strings.

Suppose R_n is a relation on set S such that s R_n t iff:

- s = t, or
- both s and t have at least n characters and the first n characters of s and t are the same.

Consider $n = 4$, then 1 R_4 1, 101 R_4 101, 1011001 R_4 10111

Section 9.5 *Practice*

Example 3: Find the equivalence class of string 1011 with respect to the equivalence relation R_4.

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So 1011 R_4 10110, 1011 R_4 10111

Section 9.5 *Practice*

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$[1011]_{R4}$ = { 10110, 10111, 101100, 101101, 101110, 101111, ... }

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Section 9.5 *Practice*

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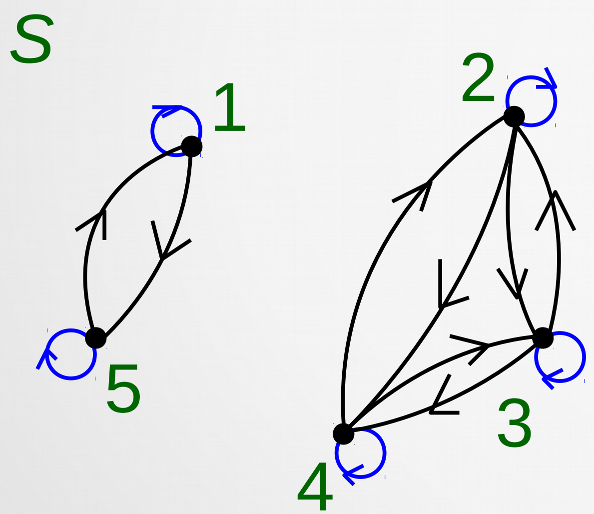
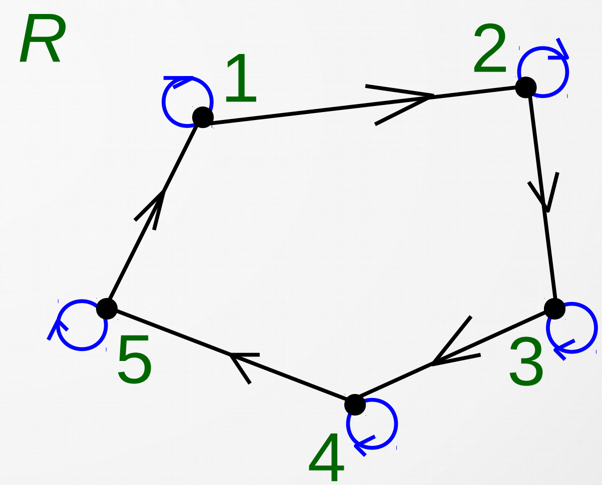
Section 9.5 *Practice*

Example 4: The relations *S* and *R* on set {1,2,3,4,5} are represented by the digraphs. Determine if any of them is an equivalence relation.





Section 9.5 *Practice*

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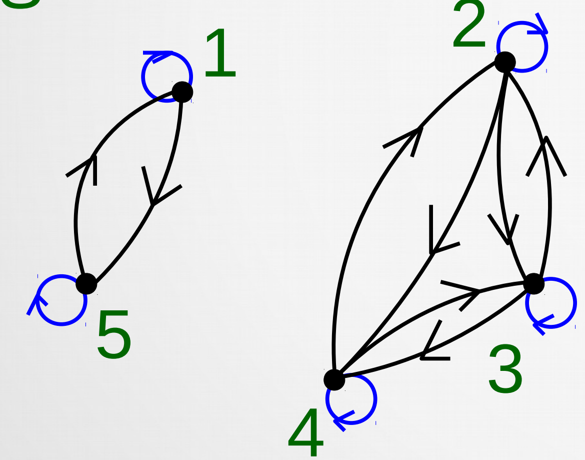
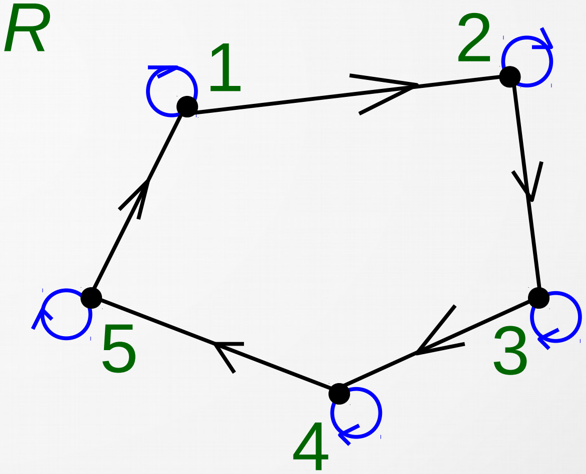


R



reflexive, *symmetric* and *transitive*, therefore, an *equivalence relation*

not symmetric, therefore, not an *equivalence relation*

Section 9.5 *Practice*

Example 5: Which of these collections of subsets are partitions of S = {-3, -2, -1, 0, 1, 2, 3} ?

a) {-3,-1,1,3}, {-2,0,2}

b) {-3,-2,-1,0}, {0,1,2,3}

c) {-3,3}, {-2,2}, {-1,1},{0}

d) {-3,-2,2,3}, {1}

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d) {-3,-2,2,3}, {1}
not a partition because their union is not S

Section 9.5 *Practice*

Example 6: Which of these collections are partitions of the set S of all bit-strings of length 10?

a) the set of bit-strings that start with 4 zeros,
the set of bit-strings that end with 6 zeros

b) the set of bit-strings that have at least 4 zeros,
the set of bit-strings that have less than 4 zeros

c) the set of bit-strings that start with 4 zeros,
the set of bit-strings that start with 4 ones,
the set of bit-strings that start with 1010,
the set of bit-strings that start with 0101

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not a partition, because string with 10 zeros belongs to both

b) the set of bit-strings that have at least 4 zeros,
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the set of bit-strings that start with 0101
not a partition, because string that starts with 1110 is not in any of the sets