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Denotations: $s\mathbf{R}t$ "s is related to t by **R**" $(s,t) \in R$

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Example:

Let **A** be a set of cities in the USA and set **B** be the set of 50 states in the USA, then let's define the relation **R** by specifying that $(a,b) \in$ **R** if a city with name a is a capital of the state b.

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Then (Albany, New York), (Trenton, New Jersey), (Hartford, Connecticut), (Harrisburg, Pennsylvania) $\in$ **R**

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Example:

Let **A**={0,1,2,3} and B={blue, red, orange}.

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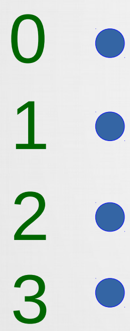

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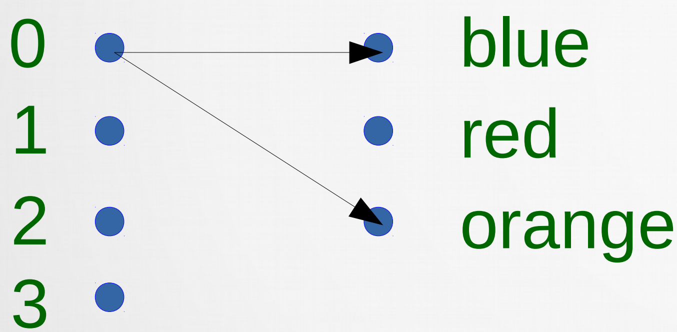

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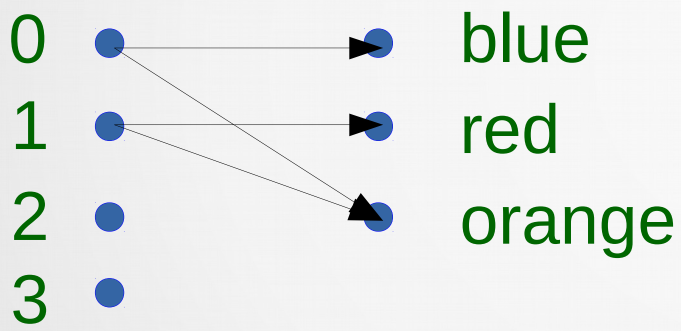

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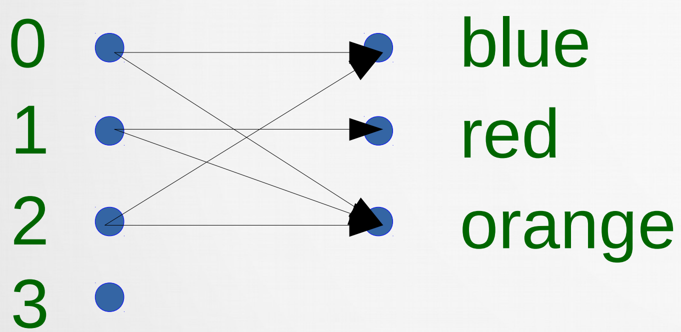

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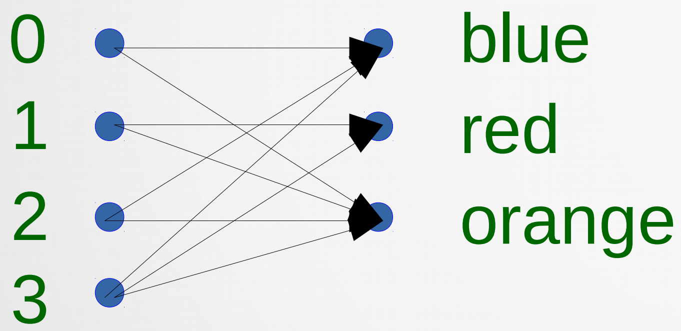

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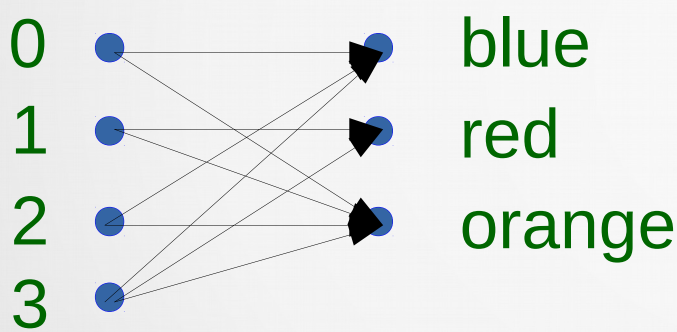

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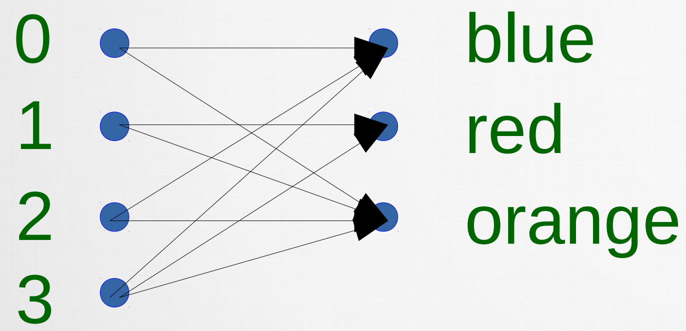

R	blue	red	orange
0			
1			
2			
3			

Relations can be represented graphically or by using a table.

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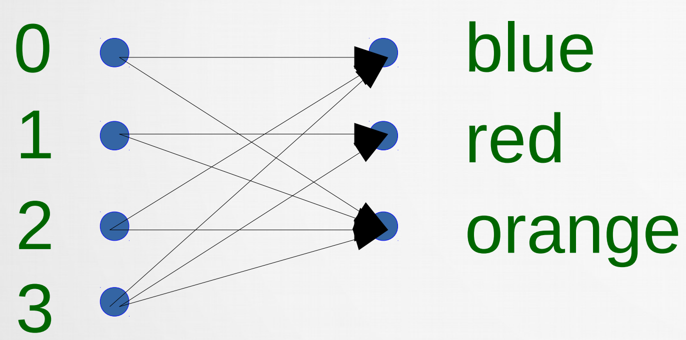

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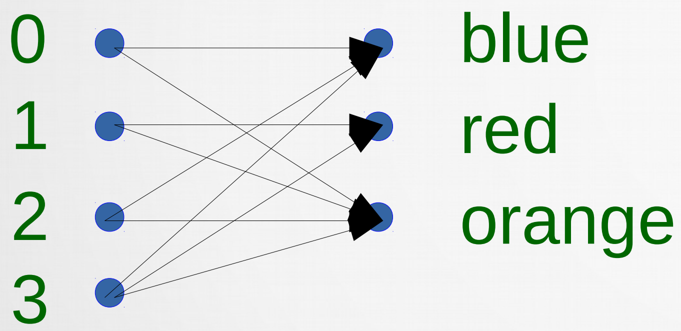

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3	×	×	×

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Functions as relations

Recall a definition of a function (we covered in CSI30):

A *function f* from a set **A** to a set **B** assigns exactly one element of **B** to each element of **A**.

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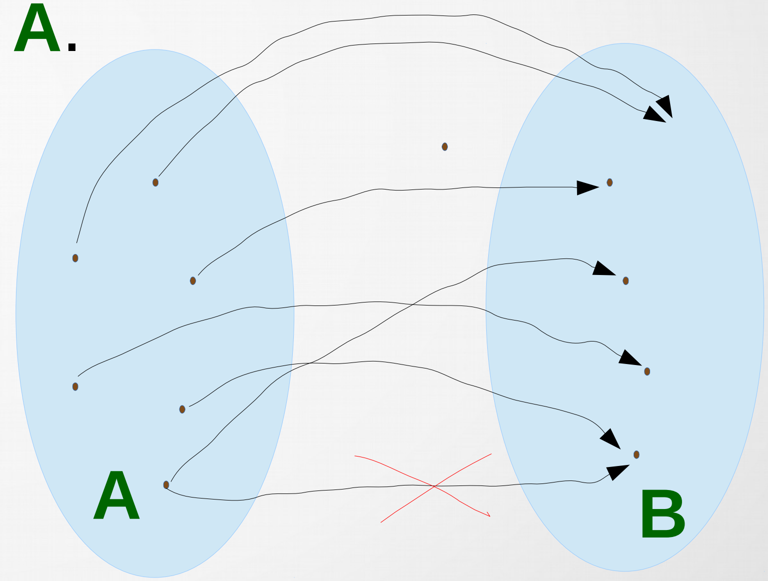

denotations:

f(a) = b

"*b* is unique element from **B** assigned to *a* from **A** by *f*"

f: A → B

"f is a function from A to B"

f is a function from ***A*** *to* ***B***

f maps **A** to **B**

names: *functions, mappings, transformations*

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A graph of f is the set of ordered pairs (a,b) such that (s.t.) $b = f(a)$.

So the graph of *f* is a subset of $\mathbf{A} \times \mathbf{B}$.

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So the graph of f is a subset of $\mathbf{A} \times \mathbf{B}$.

Therefore, we say that if a relation **R** from **A** to **B** is such that (s.t.) every element in **A** is *the first element of exactly one ordered pair* of **R**, then a function can be defined with **R** as its graph.

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Relations on sets

Relations from set A to set A are of special interest.

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How many relations are there on a set with n elements?

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Recall, that $|P(A)| = 2^{|A|}$.

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Therefore, $|P(A \times A)| = 2^{|A \times A|} = 2^{n^2}$ Answer: 2^{n^2} elements

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Example 2:

Consider the following relations on {a,b,c,d,e}:

R_1 ={(a,a),(a,c),(b,a),(b,b),(b,c),(b,d),(c,a),(c,c),(d,d),(e,e)}

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

Which of these relations are *reflexive*, which are *symmetric*, which are *antisymmetric*, and which are *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.

symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.

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Too crowded !!!

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a	x		x		
b	x	x	x	x	
c	x		x		
d				x	
e					x

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b	x	x	x	x	
c	x		x		
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b	x	x	x	x	
c	x		x		
d				x	
e					x

not symmetric

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Example 2:

Consider the following relations on {a,b,c,d,e}:

R_1 ={(a,a),(a,c),(b,a),(b,b),(b,c),(b,d),(c,a),(c,c),(d,d),(e,e)}

R_1	a	b	c	d	e
a	x		x		
b	x	x	x	x	
c	x		x		
d				x	
e					x

not antisymmetric:
we have both (a,c) and (c,a), but $a \neq c$

reflexive: $(a,a) \in R$, $\forall a \in A$.
symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.
antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.
transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_1 ={(a,a),(a,c),(b,a),(b,b),(b,c),(b,d),(c,a),(c,c),(d,d),(e,e)}

R_1	a	b	c	d	e
a	x		x		
b	x	x	x	x	
c	x		x		
d				x	
e					x

not antisymmetric:
we have both (a,c) and (c,a), but a $\neq$ c

note that *antisymmetric* is not opposite of *symmetric*, *asymmetric* is

reflexive: $(a,a) \in R$, $\forall a \in A$.
symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.
antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.
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9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_1 ={(a,a),(a,c),(b,a),(b,b),(b,c),(b,d),(c,a),(c,c),(d,d),(e,e)}

R_1	a	b	c	d	e
a	x		x		
b	x	x	x	x	
c	x		x		
d				x	
e					x

Transitive?

(a,c) ∧ (c,a) → (a,a)

reflexive: (a,a) ∈ *R* , ∀a ∈ A.
symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.
antisymmetric: (b,a) ∈ *R* ∧ (a,b) ∈ *R* → a=b, ∀a, b ∈ A.
transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_1 ={(a,a),(a,c),(b,a),(b,b),(b,c),(b,d),(c,a),(c,c),(d,d),(e,e)}

R_1	a	b	c	d	e
a	x		x x		
b	x	x	x	x	
c	x x		x		
d				x	
e					x

Transitive?

(a,c) ∧ (c,a) → (a,a)

(b,a) ∧ (a,c) → (b,c)

(b,c) ∧ (c,a) → (b,a)

(c,a) ∧ (a,c) → (c,c)

(b,d)

transitive !

reflexive: (a,a) ∈ *R* , ∀a ∈ A.

symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.

antisymmetric: (b,a) ∈ *R* ∧ (a,b) ∈ *R* → a=b, ∀a, b ∈ A.

transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

Is R_2 *reflexive*,
symmetric,
antisymmetric,
transitive?

reflexive: (a,a) ∈ *R* , ∀a ∈ A.
symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.
antisymmetric: (b,a) ∈ *R* ∧ (a,b) ∈ *R* → a=b, ∀a, b ∈ A.
transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.
symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.
antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.
transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

not *reflexive*

reflexive: (a,a) ∈ *R* , ∀a ∈ A.

symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.

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transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

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Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

reflexive: (a,a) ∈ *R* , ∀a ∈ A.
symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.
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transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

symmetric

reflexive: $(a,a) \in R$, $\forall a \in A$.

symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.

antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.

transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.

symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.

antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.

transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

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Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

not *antisymmetric*, because we have (a,b) $\wedge$ (b,a) and a $\neq$ b

reflexive: (a,a) $\in$ *R* , $\forall$a $\in$ A.
symmetric: (b,a) $\in$ *R* if (a,b) $\in$ *R*, $\forall$a, b $\in$ A.
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transitive: (a,b) $\in$ *R* $\wedge$ (b,c) $\in$ *R* $\rightarrow$ (a,c) $\in$ *R* , $\forall$a, b $\in$ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.

symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.

antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.

transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

R_2	a	b	c	d	e
a	x	x	x		
b	x	x	x		
c	x	x	x		
d				x	
e					

Is R_2 *reflexive*, *symmetric*, *antisymmetric*, *transitive*? *transitive*

(a,b) ∧ (b,a) → (a,a)

(a,b) ∧ (b,c) → (a,c)

(a,c)

(b,a) ∧ (a,b) → (b,b)

(b,a) ∧ (a,c) → (b,c)

(b,c) ∧ (c,a) → (b,a)

(c,a) ∧ (a,b) → (a,b)

(c,b) ∧ (b,a) → (c,a)

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

Is R_3 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.
symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.
antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.
transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 ***reflexive***, *symmetric*, *antisymmetric*, *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.
symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.
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9.1 Relations and their properties

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Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

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R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

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Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

symmetric

reflexive: (a,a) ∈ *R* , ∀a ∈ A.

symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.

antisymmetric: (b,a) ∈ *R* ∧ (a,b) ∈ *R* → a=b, ∀a, b ∈ A.

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9.1 Relations and their properties

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Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 *reflexive*, *symmetric*, <u>*antisymmetric*</u>, *transitive*?

reflexive: $(a,a) \in R$, $\forall a \in A$.

symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.

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transitive: $(a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

antisymmetric

reflexive: (a,a) ∈ *R* , ∀a ∈ A.

symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.

antisymmetric: (b,a) ∈ *R* ∧ (a,b) ∈ *R* → a=b, ∀a, b ∈ A.

transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

R_3	a	b	c	d	e
a	x				
b		x			
c			x		
d				x	
e					x

Is R_3 *reflexive*, *symmetric*, *antisymmetric*, *transitive*?

transitive

reflexive: (a,a) ∈ *R* , ∀a ∈ A.
symmetric: (b,a) ∈ *R* if (a,b) ∈ *R*, ∀a, b ∈ A.
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transitive: (a,b) ∈ *R* ∧ (b,c) ∈ *R* → (a,c) ∈ *R* , ∀a, b ∈ A.

9.1 Relations and their properties

Example 2:

Consider the following relations on {a,b,c,d,e}:

R_1 ={(a,a),(a,c),(b,a),(b,b),(b,c),(b,d),(c,a),(c,c),(d,d),(e,e)}

reflexive and *transitive*

R_2 ={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)}

symmetric and *transitive*

R_3 = {(a,a),(b,b),(c,c),(d,d),(e,e)}

reflexive, *symmetric*, *antisymmetric*, and *transitive*.

reflexive: $(a,a) \in R$, $\forall a \in A$.

symmetric: $(b,a) \in R$ if $(a,b) \in R$, $\forall a, b \in A$.

antisymmetric: $(b,a) \in R \wedge (a,b) \in R \rightarrow a=b$, $\forall a, b \in A$.

transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$, $\forall a, b \in A$.

9.1 Relations and their properties

Combining relations

Relations from **A** to **B** are subsets of **A** × **B**, therefore two (or more) relations can be combined!

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Example: Let **A** = {a,b,c} and **B** = {1,2,3,4},
relations R_1 = {(a,1),(a,2),(b,1),(b,3),(c,3),(c,4)} and
R_2 = {(a,1),(a,2),(a,3), (a,4), (b,4),(c,2), (c,3)}

Then

$R_1 - R_2 =$

$R_2 - R_1 =$

$R_1 \cup R_2 =$

$R_1 \cap R_2 =$

$R_1 \oplus R_2 =$

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Then

$R_1 - R_2 =$ {(b,1),(b,3),(c,4)}

$R_2 - R_1 =$

$R_1 \cup R_2 =$

$R_1 \cap R_2 =$

$R_1 \oplus R_2 =$

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Relations from **A** to **B** are subsets of **A** × **B**, therefore two (or more) relations can be combined!

Example: Let **A** = {a,b,c} and **B** = {1,2,3,4}, relations R_1 = {(a,1),(a,2),(b,1),(b,3),(c,3),(c,4)} and R_2 = {(a,1),(a,2),(a,3), (a,4), (b,4),(c,2), (c,3)}

Then

$R_1 - R_2$ = {(b,1),(b,3),(c,4)}

$R_2 - R_1$ = {(a,3), (a,4), (b,4),(c,2)}

$R_1 \cup R_2 =$

$R_1 \cap R_2 =$

$R_1 \oplus R_2 =$

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R_2 = {(a,1),(a,2),(a,3), (a,4), (b,4),(c,2), (c,3)}

Then
$R_1 - R_2$ = {(b,1),(b,3),(c,4)}
$R_2 - R_1$ = {(a,3), (a,4), (b,4),(c,2)}
$R_1 \cup R_2$ = {(a,1),(a,2),(a,3), (a,4),(b,1),(b,3),(b,4),(c,2),(c,3),(c,4)}
$R_1 \cap R_2$ =
$R_1 \oplus R_2$ =

9.1 Relations and their properties

Combining relations

Relations from **A** to **B** are subsets of **A** × **B**, therefore two (or more) relations can be combined!

Example: Let **A** = {a,b,c} and **B** = {1,2,3,4},
relations R_1 = {(a,1),(a,2),(b,1),(b,3),(c,3),(c,4)} and
R_2 = {(a,1),(a,2),(a,3), (a,4), (b,4),(c,2), (c,3)}

Then
$R_1 - R_2$ = {(b,1),(b,3),(c,4)}
$R_2 - R_1$ = {(a,3), (a,4), (b,4),(c,2)}
$R_1 \cup R_2$ = {(a,1),(a,2),(a,3), (a,4),(b,1),(b,3),(b,4),(c,2),(c,3),(c,4)}
$R_1 \cap R_2$ = {(a,1),(a,2),(c,3)}
$R_1 \oplus R_2$ =

9.1 Relations and their properties

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Relations from **A** to **B** are subsets of **A** × **B**, therefore two (or more) relations can be combined!

Example: Let **A** = {a,b,c} and **B** = {1,2,3,4},
relations R_1 = {(a,1),(a,2),(b,1),(b,3),(c,3),(c,4)} and
R_2 = {(a,1),(a,2),(a,3), (a,4), (b,4),(c,2), (c,3)}

Then
$R_1 - R_2$ = {(b,1),(b,3),(c,4)}
$R_2 - R_1$ = {(a,3), (a,4), (b,4),(c,2)}
$R_1 \cup R_2$ = {(a,1),(a,2),(a,3), (a,4),(b,1),(b,3),(b,4),(c,2),(c,3),(c,4)}
$R_1 \cap R_2$ = {(a,1),(a,2),(c,3)}
$R_1 \oplus R_2$ = {(a,3),(a,4),(b,1),(b,3),(b,4),(c,2),(c,4)} *symmetric difference*

9.1 Relations and their properties

Combining relations

Example 2:

(a) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a > b\}$, the “greater than” relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a \geq b\}$, the “greater than or equal to” relation

9.1 Relations and their properties

Combining relations

Example 2:

(a) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a > b\}$, the “greater than” relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a \geq b\}$, the “greater than or equal to” relation

Then

$R_2 - R_1 =$

9.1 Relations and their properties

Combining relations

Example 2:

(a) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a > b\}$, the “greater than” relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a \geq b\}$, the “greater than or equal to” relation

Then

$R_2 - R_1 = \{(a,b) \in \mathbf{R}^2 \mid a = b\}$

9.1 Relations and their properties

Combining relations

Example 2:

(b) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation,

and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a = b\}$, the "equal to" relation

9.1 Relations and their properties

Combining relations

Example 2:

(b) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a = b\}$, the "equal to" relation

Then

$R_1 \cap R_2 =$

9.1 Relations and their properties

Combining relations

Example 2:

(b) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a = b\}$, the "equal to" relation

Then

$R_1 \cap R_2 = \{(a,b) \in \mathbf{R}^2 \mid a = b\}$

9.1 Relations and their properties

Combining relations

Example 2:
(c) let relations
$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation, and
$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation

9.1 Relations and their properties

Combining relations

Example 2:

(c) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation

Then

$R_1 \oplus R_2 =$

9.1 Relations and their properties

Combining relations

Example 2:

(c) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation

Then

$R_1 \oplus R_2 = \{(a,b) \in \mathbf{R}^2 \mid a < b \text{ or } a > b\}$

9.1 Relations and their properties

Combining relations

Example 2:

(c) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a < b\}$, the "less than" relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a = b\}$, the "equal to" relation

Then

$R_1 \cup R_2 =$

9.1 Relations and their properties

Combining relations

Example 2:

(c) let relations

$R_1 = \{ (a,b) \in \mathbf{R}^2 \mid a < b\}$, the "less than" relation, and

$R_2 = \{ (a,b) \in \mathbf{R}^2 \mid a = b\}$, the "equal to" relation

Then

$R_1 \cup R_2 = \{(a,b) \in \mathbf{R}^2 \mid a \leq b\}$

Composite relations

Let relation ***R***: **A** → **B** and relation ***S***: **B** → **C**.

Then the composite of ***R*** and ***S*** is the relation consisting of ordered pairs (a,c), where a ∈ **A**, **c** ∈ **C**, and ∃b ∈ **B**, such that (a,b) ∈ ***R*** and (b,c) ∈ ***S***.

Denotation: ***R*** ∘ ***S***

9.1 Relations and their properties

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Example: Let relations *R* = {(1,2),(1,3),(2,3),(2,4),(3,1)} and *S* = {(2,1),(3,1),(3,2), (4,2)}. Find S ∘ R

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<u>Solution</u>: (2,1) and (1,2) produce (2,2),

9.1 Relations and their properties

Composite relations

Let relation ***R***: **A** → **B** and relation ***S***: **B** → **C**.
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Example: Let relations *R* = {(1,2),(1,3),(2,3),(2,4),(3,1)} and *S* = {(2,1),(3,1),(3,2), (4,2)}. Find S ∘ R

Solution: (2,1) and (1,2) produce (2,2),
(2,1) and (1,3) produce (2,3),

9.1 Relations and their properties

Composite relations

Let relation ***R***: **A** → **B** and relation ***S***: **B** → **C**.
Then the composite of ***R*** and ***S*** is the relation consisting of ordered pairs (a,c), where a ∈ **A**, **c** ∈ **C**, and ∃b ∈ **B**, such that (a,b) ∈ ***R*** and (b,c) ∈ ***S***.

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Example: Let relations *R* = {(1,2),(1,3),(2,3),(2,4),(3,1)} and *S* = {(2,1),(3,1),(3,2), (4,2)}. Find S ∘ R

<u>Solution</u>: (2,1) and (1,2) produce (2,2),
(2,1) and (1,3) produce (2,3),
(3,1) and (1,2),(1,3) produce (3,2) and (3,3),

9.1 Relations and their properties

Composite relations

Let relation ***R***: **A** → **B** and relation ***S***: **B** → **C**.
Then the composite of ***R*** and ***S*** is the relation consisting of ordered pairs (a,c), where a ∈ **A**, **c** ∈ **C**, and ∃b ∈ **B**, such that (a,b) ∈ ***R*** and (b,c) ∈ ***S***.

Denotation: ***R*** ∘ ***S***

Example: Let relations *R* = {(1,2),(1,3),(2,3),(2,4),(3,1)} and *S* = {(2,1),(3,1),(3,2), (4,2)}. Find S ∘ R

<u>Solution</u>: (2,1) and (1,2) produce (2,2),
(2,1) and (1,3) produce (2,3),
(3,1) and (1,2),(1,3) produce (3,2) and (3,3),
(3,2) and (2,3),(2,4) produce (3,3) and (3,4),

9.1 Relations and their properties

Composite relations

Let relation ***R***: **A** → **B** and relation ***S***: **B** → **C**.
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Example: Let relations *R* = {(1,2),(1,3),(2,3),(2,4),(3,1)} and *S* = {(2,1),(3,1),(3,2), (4,2)}. Find S ∘ R

<u>Solution</u>: *(2,1) and (1,2) produce (2,2),*
(2,1) and (1,3) produce (2,3),
(3,1) and (1,2),(1,3) produce (3,2) and (3,3),
(3,2) and (2,3),(2,4) produce (3,3) and (3,4),
(4,2) and (2,3),(2,4) produce (4,3) and (4,4)

9.1 Relations and their properties

Composite relations

Let relation ***R***: **A** → **B** and relation ***S***: **B** → **C**.
Then the composite of ***R*** and ***S*** is the relation consisting of ordered pairs (a,c), where a ∈ **A**, **c** ∈ **C**, and ∃b ∈ **B**, such that (a,b) ∈ ***R*** and (b,c) ∈ ***S***.

Denotation: ***R*** ∘ ***S***

Example: Let relations *R* = {(1,2),(1,3),(2,3),(2,4),(3,1)} and *S* = {(2,1),(3,1),(3,2), (4,2)}. Find S ∘ R

Answer: S ∘ R = { (2,2), (2,3), (3,2), (3,3), (3,3), (3,4), (4,3), (4,4)}

9.1 Relations and their properties

Composing the parent relation with itself

Let $\boldsymbol{R}$ be a relation on set **A**. The powers $\boldsymbol{R^n}$, *n=1,2,3,...* are defined recursively by

$$R^1 = R$$
$$R^{n+1} = R^n \circ R$$

9.1 Relations and their properties

Composing the parent relation with itself

Let R be a relation on set **A**. The powers R^n , $n=1,2,3,\ldots$ are defined recursively by

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Example: The relation $R = \{(1,2),(1,3),(2,3),(2,4),(3,1)\}$. Find the powers R^n, where $n=2,3,4$

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<u>*Solution*</u>: $R^2 = R \circ R =$

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(1,2) and (2,3),(2,4) produce (1,3) and (1,4)

(1,3) and (3,1) produce (1,1)

(2,3) and (3,1) produce (2,1)

(3,1) and (1,2),(1,3) produce (3,2) and (3,3)

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(1,2) and (2,3),(2,4) produce (1,3) and (1,4)

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<u>Solution</u>: $R^2 = R \circ R = \{(1,1), (1,3), (1,4), (2,1), (3,2), (3,3)\}$
$R^3 = R^2 \circ R =$
(1,1) and (1,2), (1,3) produce (1,2),(1,3)

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<u>Solution</u>: $R^2 = R \circ R = \{(1,1), (1,3), (1,4), (2,1), (3,2), (3,3)\}$

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$R^3 = R^2 \circ R =$

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(1,3) and (3,1) produce (1,1) *(1,4) doesn't have a pair*

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$R^3 = R^2 \circ R =$

(1,1) and (1,2), (1,3) produce (1,2),(1,3)

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$R^3 = R^2 \circ R =$

(1,1) and (1,2), (1,3) produce (1,2),(1,3)

(1,3) and (3,1) produce (1,1) *(1,4) doesn't have a pair*

(2,1) and (1,2), (1,3) produce (2,2),(2,3)

(3,2) and (2,3),(2,4) produce (3,3),(3,4)

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Composing the parent relation with itself

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Example: The relation $R = \{(1,2),(1,3),(2,3),(2,4),(3,1)\}$. Find the powers R^n, where $n=2,3,4$

<u>Solution</u>: $R^2 = R \circ R = \{(1,1), (1,3), (1,4), (2,1), (3,2), (3,3)\}$
$R^3 = R^2 \circ R =$
(1,1) and (1,2), (1,3) produce (1,2),(1,3)
(1,3) and (3,1) produce (1,1) *(1,4) doesn't have a pair*
(2,1) and (1,2), (1,3) produce (2,2),(2,3)
(3,2) and (2,3),(2,4) produce (3,3),(3,4)
(3,3) and (3,1) produce (3,1)

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Solution: $R^2 = R \circ R = \{(1,1), (1,3), (1,4), (2,1), (3,2), (3,3)\}$
$R^3 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (3,4), (3,2), (3,4)\}$
(1,1) and (1,2), (1,3) produce (1,2),(1,3)
(1,3) and (3,1) produce (1,1) *(1,4) doesn't have a pair*
(2,1) and (1,2), (1,3) produce (2,2),(2,3)
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9.1 Relations and their properties

Composing the parent relation with itself

Let $\boldsymbol{R}$ be a relation on set **A**. The powers $\boldsymbol{R^n}$, $n=1,2,3,...$ are defined recursively by

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9.1 Relations and their properties

Composing the parent relation with itself

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$$R^{n+1} = R^n \circ R$$

Example: The relation $R = \{(1,2),(1,3),(2,3),(2,4),(3,1)\}$. Find the powers R^n, where $n=2,3,4$

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Answer:

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