Strong induction Practice

Example 1:

(Rosen, №6, page 342)

a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.

b) Prove your answer to a) using the principle of mathematical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.

c) Prove your answer to a) using strong induction. How does the inductive hypothesis in this proof differ from that in the b)?

Strong induction Practice

Example 1:

(Rosen, №6, page 342)

a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.

$3c$, $2 \times 3c = 6c$, $3 \times 3c = 9c$, $10c$, $4 \times 3c = 12c$,
$3c + 10c = 13c$, $5 \times 3c = 15c$, $2 \times 3c + 10c = 16c$,
$6 \times 3c = 18c$, $3 \times 3c + 10c = 19c$, $2 \times 10c = 20c$,
$7 \times 3c = 21c$, $4 \times 3c + 10c = 22c$, …

Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.
$P(n)$, where $n \geq 18$

Strong induction Practice

Example 1:

(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

P(n), where $n \geq 18$

b) Proof by math. induction:

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Example 1:

(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

P(n), where $n \geq 18$

b) Proof by math. induction:

Basis step: P(18) = $6 \times 3c$ = 18c,

P(19) = $3 \times 3c + 10c$ = 19c,

P(20) = $2 \times 10c$ = 20c

– it is enough, because next case will be $7 \times 3c = 21c$

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Example 1:

(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

b) Proof by math. induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(k)$ is true. IH

Let's show that in this case $P(k+1)$ is also true.

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Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(k)$ is true. IH

Let's show that in this case $P(k+1)$ is also true.

If k-cents postage:

- consists of only 3-cent stamps, then there are more that three of them ($k \geq 20$), so replace the three 3-cent stamps with one 10-cent stamp

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a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

b) Proof by math. induction:
Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(k)$ is true. IH
Let's show that in this case $P(k+1)$ is also true.
If k-cents postage:

- consists of only 3-cent stamps, then there are more that three of them ($k \geq 20$), so replace the three 3-cent stamps with one 10-cent stamp
- includes at least two 10-cent stamp, replace two 10-cent stamps with seven 3-cent stamps

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b) Proof by math. induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(k)$ is true. IH

Let's show that in this case $P(k+1)$ is also true. If k-cents postage:

- includes at least one 10-cent stamp and three 3-cent stamps, replace one 10-cent stamps and three 3-cent stamps with two 10-cent stamps

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a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

b) Proof by math. induction:

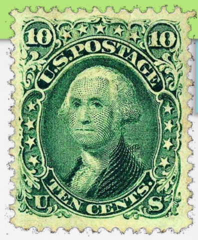
Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(k)$ is true. IH

Let's show that in this case $P(k+1)$ is also true. If k-cents postage:

- includes at least one 10-cent stamp and three 3-cent stamps, replace one 10-cent stamps and three 3-cent stamps with two 10-cent stamps
  - this covers all the possible cases

We showed that $P(k+1)$ is also true. This completes the ind. step.

Strong induction Practice



Example 1:
(Rosen, №6, page 342)
a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

b) Proof by math. induction:
Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(k)$ is true. IH
...
We showed that P(k+1) is also true. This completes the ind. Step.
By mathematical induction we proved that any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

q.e.d.

Strong induction Practice

Example 1:

(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

P(n), where $n \geq 18$

c) Proof by strong induction:

Basis step: $P(18) = 6 \times 3c = 18c$,

$P(19) = 3 \times 3c + 10c = 19c$,

$P(20) = 2 \times 10c = 20c$

– it is enough, because next case will be $7 \times 3c = 21c$

Strong induction Practice

Example 1:

(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

P(n), where $n \geq 18$

c) Proof by strong induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(j)$ is true for any integer j, $18 \leq j \leq k$, i.e.

$P(18) \wedge P(19) \wedge \ldots \wedge P(k)$ is true IH

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P(n), where $n \geq 18$

c) Proof by strong induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(18) \wedge P(19) \wedge \ldots \wedge P(k)$ is true IH

Let's show that in this case $P(k+1)$ is also true:

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P(n), where $n \geq 18$

c) Proof by strong induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$,

$P(18) \wedge P(19) \wedge \ldots \wedge P(k)$ is true IH

Let's show that in this case $P(k+1)$ is also true:

Since $k-2 \geq 18$, we can form a postage of $(k-2)$- cents

$P(k-2)$,

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a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.
P(n), where $n \geq 18$

c) Proof by strong induction:
Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(18) \wedge P(19) \wedge \ldots \wedge P(k)$ is true IH
Let's show that in this case P(k+1) is also true:
Since $k-2 \geq 18$, we can form a postage of (k-2)- cents P(k-2), and then by adding a 3-cent stamp we will reach (k+1)-cent postage. Hence P(k+1) is also true.

Strong induction Practice

Example 1:

(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

P(n), where n ≥ 18

c) Proof by strong induction:

Inductive step: assume that for any arbitrary fixed k ≥ 20,

P(18) ∧ P(19) ∧ … ∧ P(k) is true IH

Let's show that in this case P(k+1) is also true:

Since k-2 ≥ 18, we can form a postage of (k-2)- cents P(k-2), and then by adding a 3-cent stamp we will reach (k+1)-cent postage. Hence P(k+1) is also true.

This completes the inductive step.

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Example 1:
(Rosen, №6, page 342)
a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.
$P(n)$, where $n \geq 18$

c) Proof by strong induction:
Inductive step: assume that for any arbitrary fixed $k \geq 20$,
$P(18) \wedge P(19) \wedge \ldots \wedge P(k)$ is true IH
Let's show that in this case $P(k+1)$ is also true:
Since $k-2 \geq 18$, we can form a postage of $(k-2)$- cents $P(k-2)$, and then by adding a 3-cent stamp we will reach $(k+1)$-cent postage. Hence $P(k+1)$ is also true.
This completes the inductive step.
By strong induction, for any $n \geq 18$, $P(n)$ is true.

q.e.d.

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Example 1:
(Rosen, №6, page 342)

a) Any postage of 18 or more cents can be formed using just 3-cent and 10-cent stamps.

P(n), where $n \geq 18$

b) Proof by math. induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$, P(k) is true. IH

c) Proof by strong induction:

Inductive step: assume that for any arbitrary fixed $k \geq 20$, $P(18) \wedge P(19) \wedge \ldots \wedge P(k)$ is true IH