Mathematical induction: Practice

Example 1: Find the formula for

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots\frac{1}{n(n+1)}$$

by examining the values of this expression for small values of *n*. Then prove it!

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Example 1: Find the formula for $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots\frac{1}{n(n+1)}$

by examining the values of this expression for small values of *n*. Then prove it!

a) obviously n $\neq$ 0
when n = 1 we get $\frac{1}{1\cdot 2}=\frac{1}{2}$, when n = 2 we get $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}=\frac{4}{6}=\frac{2}{3}$, when n = 3 we get $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}=\frac{9}{12}=\frac{3}{4}$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots$

n = 1, n = 2, n = 3, n = 4, n = 5

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots\frac{1}{n(n+1)}=\frac{n}{n+1} \quad \text{or} \quad \sum_{i=1}^{n}\frac{1}{i(i+1)}=\frac{n}{n+1}$$ for any positive integer n

b) proof (by math. Induction):

P(n): $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots\frac{1}{n(n+1)}=\frac{n}{n+1}$ Let's prove that for any positive integer n P(n) is true

Base step: P(1): $\frac{1}{1\cdot 2}=\frac{1}{2}$ true. This completes base step.

Inductive step: let P(k) be true for any arbitrary fixed k $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots\frac{1}{k(k+1)}=\frac{k}{k+1}$ (IH)

Let's prove that under this assumption P(k+1) will be also true

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots\frac{1}{k(k+1)}+\frac{1}{(k+1)((k+1)+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$$

Hence P(k+1) is also true (under the inductive hypotheses). This completes the inductive step.
Therefore by math. induction P(n) is true for any positive integer n. qed

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Example 2: Prove that

$$\sum_{j=0}^{n}\left(-\frac{1}{2}\right)^{j}=\frac{2^{n+1}+(-1)^{n}}{3\cdot 2^{n}}$$

whenever *n* is a nonnegative integer

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Example 2: Prove that $\sum_{j=0}^{n}\left(-\frac{1}{2}\right)^j=\frac{2^{n+1}+(-1)^n}{3\cdot 2^n}$ whenever *n* is a non-negative integer

Proof:Let P(n) be $\sum_{j=0}^{n}\left(-\frac{1}{2}\right)^j=\frac{2^{n+1}+(-1)^n}{3\cdot 2^n}$. Let's prove that for any non-negative integer n P(n) is true.

Base step: n = 0 $\sum_{j=0}^{0}\left(-\frac{1}{2}\right)^j=1=\frac{2^{0+1}+(-1)^0}{3\cdot 2^0}=\frac{2+1}{3\cdot 1}=1$ we can see that P(0) is true

Inductive step: let's assume that P(k) is true for any arbitrary fixed non-negative integer. (IH)

P(k): $\sum_{j=0}^{k}\left(-\frac{1}{2}\right)^j=\frac{2^{k+1}+(-1)^k}{3\cdot 2^k}$

Let's prove that P(k+1) is also true (under this assumption)

P(k+1):

$$\sum_{j=0}^{k+1}\left(-\frac{1}{2}\right)^j=\sum_{j=0}^{k}\left(-\frac{1}{2}\right)^j+\left(-\frac{1}{2}\right)^{k+1}=\frac{2^{k+1}+(-1)^k}{3\cdot 2^k}+\left(-\frac{1}{2}\right)^{k+1}=\frac{2^{k+1}+(-1)^k}{3\cdot 2^k}+\left((-1)^{k+1}\frac{1^{k+1}}{2^{k+1}}\right)=\frac{2\left(2^{k+1}+(-1)^k\right)}{3\cdot 2^{k+1}}+\frac{3(-1)^{k+1}1^{k+1}}{3\cdot 2^{k+1}}=\dots$$

$$\dots=\frac{2^{k+2}+2(-1)^k+3(-1)^{k+1}}{3\cdot 2^{k+1}}=\frac{2^{k+2}+(-1)^k(2+3(-1))}{3\cdot 2^{k+1}}=\frac{2^{k+2}+(-1)^k(-1)}{3\cdot 2^{k+1}}=\frac{2^{k+2}+(-1)^{k+1}}{3\cdot 2^{k+1}}$$

Therefore P(k+1) is also true (under the inductive hypotheses). This completes the inductive step.

Hence P(n) is true for any non-negative integer n

qed

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Example 3: Prove that $3^n < n!$ for integer $n > 6$

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Example 3: Prove that $3^n < n!$ for integer n > 6

Proof:Let P(n) be $3^n < n!$ Let's prove that for any integer n>6, P(n) is true.

Base step: n = 7 : $3^7 = 2187$, $7! = 1 * 2 * 3 * 4 * 5 * 6 * 7 = 11340$, thus $3^7 < 7!$
we can see that P(7) is true

Inductive step: let's assume that P(k) is true for any arbitrary fixed integer n>6. (IH)

P(k): $3^k < k!$

Let's prove that P(k+1) is also true (under this assumption)

P(k+1): 3^{k+1} ? $(k+1)!$
$3^k * 3$? $k! * k$
We know that $3^k < k!$
in addition $3 < k$ because k > 6
Therefore $3^k * 3 < k! * k$
Hence P(k+1) is also true (under the inductive hypotheses). This completes the inductive step.

By mathematical induction, we can conclude that P(n) is true for any non-negative integer n

qed