Section 5.1 Mathematical Induction

There are many mathematical statements that assert a property for all positive integers.

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$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$ or $$\sum_{i=1}^{n} i=\frac{n(n+1)}{2}$$

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Can be proven with mathematical induction

Section 5.1 Mathematical Induction

Principle of mathematical induction:

Assume that $P(n)$ is a propositional function. To prove that $P(n)$ is true for all positive integers n we complete two steps:

BASIS STEP (BASE): We verify that $P(1)$ is true

note: it is not always 1

INDUCTIVE STEP: We show that the conditional statement $P(k) \rightarrow P(k+1)$ is true for all positive integers k.

to do the inductive step: we assume that $P(k)$ is true for an arbitrary positive k and show that under this assumption $P(k+1)$ must also be true.

Review: Functions

Let $f(x) = 2x+3$
What is $f(x+1)$?

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Let $g(x) = 3x - 4x^2 + 19$

What is $g(x-1)$?

$$g(x-1) = 3(x-1) - 4(x-1)^2 + 19 =$$
$$= 3x - 3 - 4(x^2-2x+1) + 19 =$$
$$= 3x - 3 - 4x^2 + 8x - 4 + 19 =$$
$$= -4x^2 + 11x + 12$$

Section 5.1 Mathematical Induction

Example: Let's prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, i.e.
the sum of the first n positive integers $1+2+3+\ldots+n=\frac{n(n+1)}{2}$

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$$P(k) = 1+2+3+\ldots+k = \frac{k(k+1)}{2}$$

We need to show that $P(k+1) = \frac{(k+1)((k+1)+1)}{2} = \frac{(k+1)(k+2)}{2}$

Video link: https://www.khanacademy.org/math/algebra-home/alg-series-and-induction/alg-induction/v/proof-by-induction

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$$P(k)=1+2+3+\ldots+k=\frac{k(k+1)}{2}$$

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pulling out last term

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Section 5.1 Mathematical Induction

Visualizations of mathematical induction:

climbing an infinite ladder

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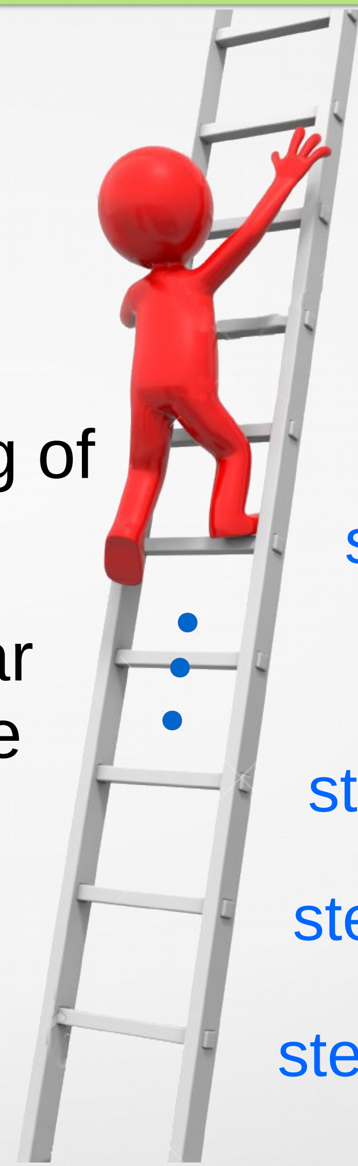
Visualizations of mathematical induction:

climbing an infinite ladder

- we can reach the first rung of the ladder   P(1)
- If we can reach a particular rung of the ladder, then we can reach the next rung

  $P(k) \rightarrow P(k+1)$

Base: we can reach step 1



P(n) – we can reach nth rung

step k+1

step k

step 3

step 2

step 1

Section 5.1 Mathematical Induction

<u>Visualizations of mathematical induction</u>:

climbing an infinite ladder

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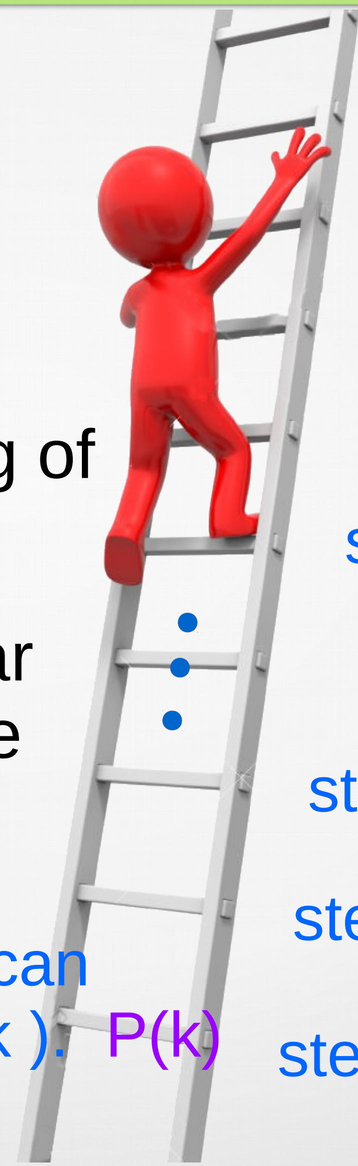
- If we can reach a particular rung of the ladder, then we can reach the next rung

$P(k) \rightarrow P(k+1)$

Induction step: assume that we can reach k^{th} rung (for any arbitrary k). $P(k)$



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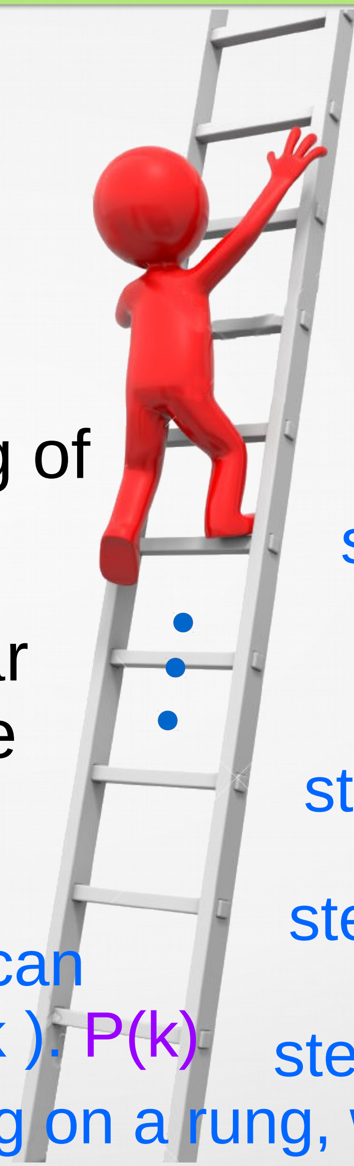

$P(n)$ – we can reach n^{th} rung

step k+1

step k

step 3

step 2

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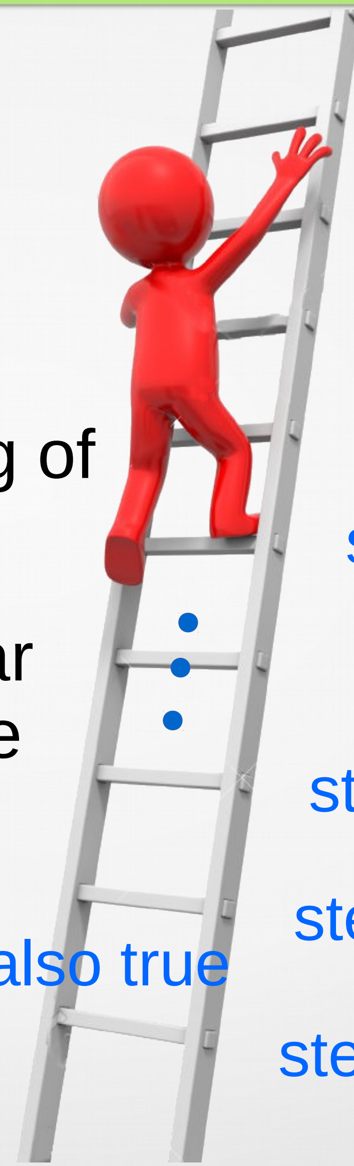
Induction step: assume that we can reach k^{th} rung (for any arbitrary k). $P(k)$

We also know that when standing on a rung, we can reach next rung.

Section 5.1 Mathematical Induction

<u>Visualizations of mathematical induction</u>:

climbing an infinite ladder



P(n) – we can reach n^{th} rung

step k+1

step k

step 3

step 2

step 1

- we can reach the first rung of the ladder P(1)
- If we can reach a particular rung of the ladder, then we can reach the next rung

$P(k) \rightarrow P(k+1)$

Induction step: hence P(k+1) is also true

Section 5.1 Mathematical Induction

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climbing an infinite ladder

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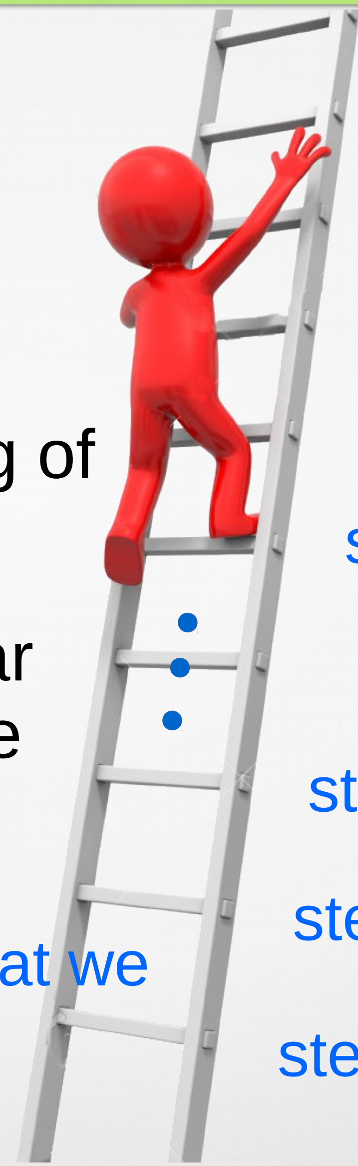
- If we can reach a particular rung of the ladder, then we can reach the next rung

$P(k) \rightarrow P(k+1)$

By math. induction we proved that we can climb an infinite ladder

qed



P(n) – we can reach n^{th} rung

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Section 5.1 Mathematical Induction

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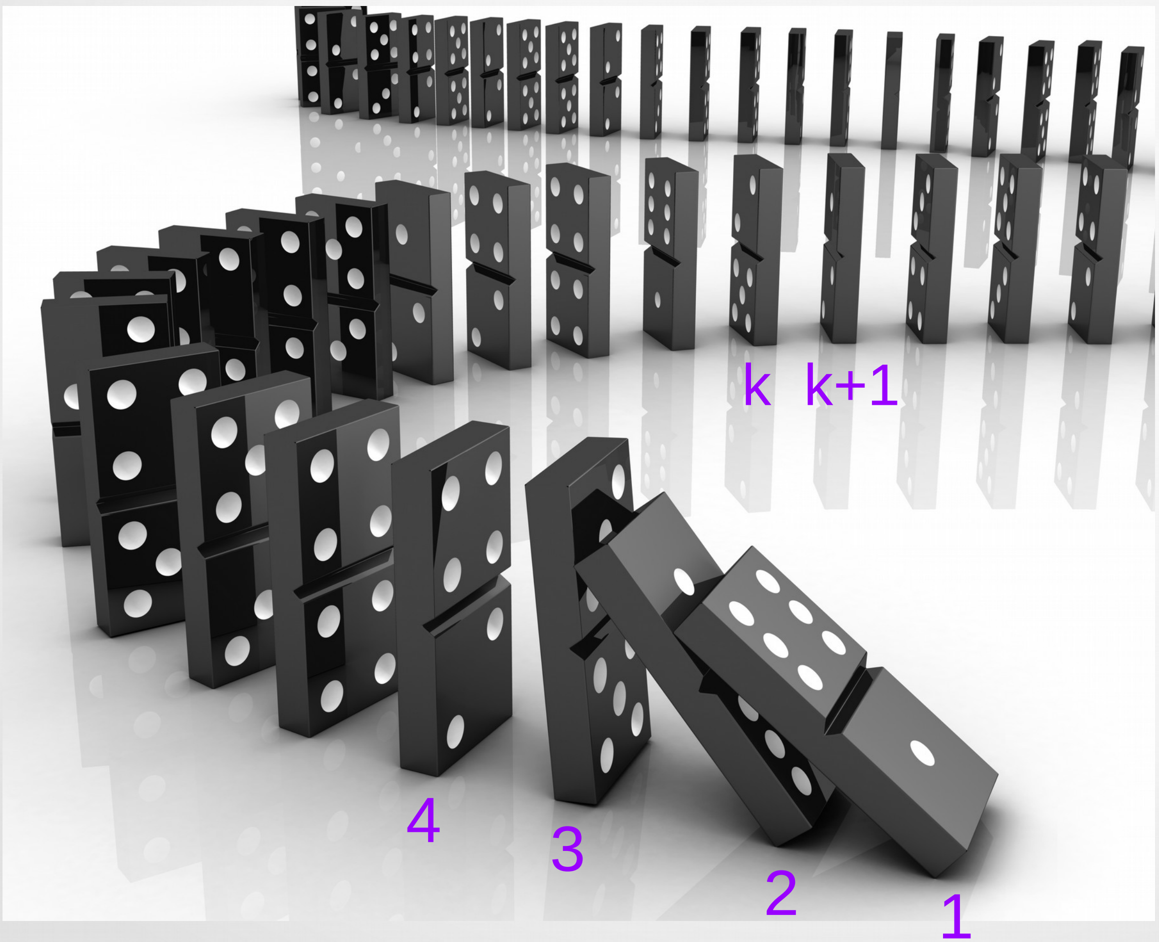

$P(n)$ – domino n is knocked over

we can knock over the 1st domino $P(1)$

If k^{th} domino is knocked it knocks over the next domino $(k+1)^{th}$
$P(k) \rightarrow P(k+1)$

Section 5.1 Mathematical Induction

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Proof (by contradiction): assume that $P(1)$ it true, and $P(k) \rightarrow P(k+1)$ is true for all positive integers k.
We need to show that in this case $P(n)$ is true for all positive integers n.

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Assume it is false.

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Assume it is false. i.e. math induction doesn't work (proof by contradiction).

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In this case there is at least one positive integer i for which P(i) is false.
Let S be the set of positive integers for which P(n) is false.

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$\exists\, i \in Z^+$ ($P(i)$ is false)

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Set S has a least element according to The Well-Ordering Property, let's name it m. $P(m)$ is false.

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$m \neq 1$ because P(1) it true, hence $m > 1$

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m-1 is a positive integer, therefore P(m-1) must be true (m is the smallest where P(n) fails).

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~~$\exists i \in Z^+ (P(i)$ is false)~~ Our assumption was false!

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We need to show that in this case $P(n)$ is true for all positive integers n.
~~Assume it is false.~~ i.e. math induction doesn't work (proof by contradiction) Our assumption was false!

So there is no positive integer at which $P(n)$ fails.
Therefore $P(n)$ is true for all positive integers n.
q.e.d. (Quod Erat Demonstrandum)

that which was to be demonstrated

Section 5.1 Mathematical Induction

The good and the bad of mathematical induction

good: we can prove a conjecture (statement) once it is made and is true.

bad: math. induction cannot be used to find new theorems

proofs by math. induction do not provide insights as to why theorems are true

Section 5.1 Mathematical Induction

Example 1: Let P(n) be the statement that

$$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

for any positive integer *n*.

(*Rosen, p. 329 № 4*)

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the IH (Inductive Hypotheses): assume that P(k) is true for an arbitrary fixed integer k ≥ 1

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$$\frac{k^2(k+1)^2}{4}+(k+1)^3=(k+1)^2\left(\frac{k^2}{4}+(k+1)\right)=(k+1)^2\left(\frac{k^2+4k+4}{4}\right)=\ldots$$

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$$(k+1)^2\frac{(k+2)^2}{4}=\left(\frac{(k+1)(k+2)}{2}\right)^2$$

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Let's prove that P(k+1) is also true:

$$1^3+2^3+3^3+\ldots+k^3+(k+1)^3=\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3=\ldots$$

$$\frac{k^2(k+1)^2}{4}+(k+1)^3=(k+1)^2\left(\frac{k^2}{4}+(k+1)\right)=(k+1)^2\left(\frac{k^2+4k+4}{4}\right)=\ldots$$

$$(k+1)^2\frac{(k+2)^2}{4}=\left(\frac{(k+1)(k+2)}{2}\right)^2$$

← shows that P(k+1) is true under the IH.

Section 5.1 Mathematical Induction

Example 1: Let P(n) be the statement that

$$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

Proof:

2) Inductive step:

the IH: assume that P(k) is true for an arbitrary fixed integer k ≥ 1

$$1^3+2^3+3^3+\ldots+k^3=\left(\frac{k(k+1)}{2}\right)^2$$

Let's prove that P(k+1) is also true:

$$1^3+2^3+3^3+\ldots+k^3+(k+1)^3=\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3=\ldots$$

$$\frac{k^2(k+1)^2}{4}+(k+1)^3=(k+1)^2\left(\frac{k^2}{4}+(k+1)\right)=(k+1)^2\left(\frac{k^2+4k+4}{4}\right)=\ldots$$

$$(k+1)^2\frac{(k+2)^2}{4}=\left(\frac{(k+1)(k+2)}{2}\right)^2$$

← shows that P(k+1) is true under the IH.

This completes the inductive step

Section 5.1 Mathematical Induction

Example 1: Let P(n) be the statement that

$$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

(Rosen, p. 329 № 4)

for any positive integer *n*.

Therefore, by mathematical induction P(n) is true for all positive integers n.

qed