In-class Practice

1. Given a sequence $\{a_k\}$, beginning with a_1:

10, 17, 24, 31, 38, 45

a) what is a_3?

b) What is the index of the term 38 in the sequence?

c) What is the *final index* of the sequence?

d) Is the sequence *increasing*, *decreasing*, *non-increasing*, *non-decreasing*?

2. Consider the sequence defined by the formula

$$b_k = k^2 \text{ for } k \geq 2.$$

What is the third term in the sequence?

In-class Practice

1. Given a sequence $\{a_k\}$, beginning with a_1:

10, 17, 24, 31, 38, 45

a) what is a_3? 24

b) What is the index of the term 38 in the sequence? 5

c) What is the *final index* of the sequence? 6

d) Is the sequence *increasing*, *decreasing*, *non-increasing*, *non-decreasing*? increasing

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$$b_k = k^2 \text{ for } k \geq 2.$$

What is the third term in the sequence?

4, 9, 16, 25, ...

In-class Practice

3. Let $\{s_n\}$ be an arithmetic sequence that starts with an initial index of 0. The *initial term* is 3 and the *common difference* is -2. What is s_2? What is s_{12}?

4. Consider the arithmetic sequence: 7, 4, 1, ...

a) What is the *initial term* of the sequence?

b) What is the *common difference*?

c) What is the next term in the sequence?

5. Consider the sequence defined by the formula

$$t_k = (-1)^k \frac{1}{k} \qquad \text{for } k \geq 2.$$

List the first 5 terms of the sequence?

In-class Practice

3. Let $\{s_n\}$ be an arithmetic sequence that starts with an initial index of 0. The *initial term* is 3 and the *common difference* is -2. What is s_2? What is s_{12}?

$s_0 = 3$,

$s_1 = 3 + (-2) = 1$,

$s_2 = 1 + (-2) = -1$,

...

$s_n = s_0 + dn$,

hence $s_{12} = 3 + (-2) \times 12 = -21$

Answer: $s_2 = -1$, $s_{12} = -21$

In-class Practice

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$$t_2 = (-1)^2 \frac{1}{2} = \frac{1}{2}, t_3 = (-1)^3 \frac{1}{3} = -\frac{1}{3}, t_4 = (-1)^4 \frac{1}{4} = \frac{1}{4}, t_5 = -\frac{1}{5}, t_6 = \frac{1}{6},$$

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$$t_k = (-1)^k \frac{1}{k} \qquad \text{for } k \geq 2.$$

Answer: $\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}$

In-class Practice

6. Let $\{s_n\}$ be a geometric sequence that starts with an initial index of 0. The *initial term* is 2 and the *common ratio* is 5. What is s_2?

7. Let $\{s_n\}$ be a geometric sequence that starts with an *initial index* of 0. The *initial term* is 16 and the common ratio is $\frac{1}{2}$. What is s_3?

8. Let $\{s_n\}$ be a geometric sequence that starts with an *initial index* of 0. The *initial term* is 1 and the common ratio is $-\frac{1}{3}$. What is s_4? What is s_{14}?

In-class Practice

6. Let $\{s_n\}$ be a geometric sequence that starts with an initial index of 0. The *initial term* is 2 and the *common ratio* is 5. What is s_2?

$s_0 = 2$

$s_1 = 2 \cdot 5 = 10$

$s_2 = 10 \cdot 5 = 50$

Answer: $s_2 = 50$

In-class Practice

6. Let $\{s_n\}$ be a geometric sequence that starts with an initial index of 0. The *initial term* is 2 and the *common ratio* is 5. What is s_2?

Answer: $s_2 = 50$

7. Let $\{s_n\}$ be a geometric sequence that starts with an *initial index* of 0. The *initial term* is 16 and the common ratio is $\frac{1}{2}$. What is s_3?

$s_0 = 16$

$s_1 = 16 \cdot \frac{1}{2} = 8$

$s_2 = 8 \cdot \frac{1}{2} = 4$

$s_3 = 4 \cdot \frac{1}{2} = 2$

or use $s_n = s_0 \cdot r^n$

$s_3 = 16 \cdot \left(\frac{1}{2}\right)^3 = 2$

In-class Practice

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In-class Practice

$s_0 = 1$ we will use formula $s_n = s_0 \cdot r^n$

$$s_4 = 1\times\left(-\frac{1}{3}\right)^4 = \frac{1}{81}$$

$$s_{14} = 1\times\left(-\frac{1}{3}\right)^{14} = \frac{1}{3^{14}}$$

Answer: $s_4 = \frac{1}{81}$, $s_{14} = \frac{1}{3^{14}}$

8. Let $\{s_n\}$ be a geometric sequence that starts with an *initial index* of 0. The *initial term* is 1 and the common ratio is $-\frac{1}{3}$. What is s_4? What is s_{14}?

In-class Practice

9. Compute $\sum_{i=-1}^{4}\left(i^2-4\right)$

10. Re-write the given summation with the final term in the sum removed

$$\sum_{i=0}^{n} 3^{i+1}$$

11. Find $\sum_{k=0}^{156} k$

12. Find $\sum_{k=13}^{178} k$

13. Find $\sum_{k=0}^{6} 3^k$

14. Find $\sum_{k=-1}^{20}\left(3+5k\right)$

In-class Practice

9. Compute $\sum_{i=-1}^{4} (i^2 - 4)$

$$\sum_{i=-1}^{4} (i^2-4) = ((-1)^2-4)+(0^2-4)+(1^2-4)+(2^2-4)+(3^2-4)+(4^2-4) = \ldots$$

$$\ldots = (-3)+(-4)+(-3)+0+5+12 = 7$$

Answer: $\sum_{i=-1}^{4} (i^2-4) = 7$

In-class Practice

9. Compute $\sum_{i=-1}^{4}(i^2-4)$ Answer: $\sum_{i=-1}^{4}(i^2-4)=7$

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10. Re-write the given summation with the final term in the sum removed

$$\sum_{i=0}^{n} 3^{i+1}=\sum_{i=0}^{n-1} 3^{i+1}+3^{n+1}$$

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11. Find $\sum_{k=0}^{156} k$

$$\sum_{k=0}^{156} k=\frac{(0+156)157}{2}=12{,}246$$

last term

number of terms

first term

$$\sum_{i=0}^{n-1} a_i=\left(a_0+a_{n-1}\right)\frac{n}{2}$$

In-class Practice

9. Compute $\sum_{i=-1}^{4}(i^2-4)$ Answer: $\sum_{i=-1}^{4}(i^2-4)=7$

10. Re-write the given summation with the final term in the sum removed

$$\sum_{i=0}^{n}3^{i+1}$$

Answer: $\sum_{i=0}^{n}3^{i+1}=\sum_{i=0}^{n-1}3^{i+1}+3^{n+1}$

11. Find $\sum_{k=0}^{156}k=12,246$

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12. Find $\sum_{k=13}^{178}k$

1st way:

$$\sum_{k=13}^{178}k=\frac{(13+178)(178-13+1)}{2}=15{,}853$$

In-class Practice

9. Compute $\sum_{i=-1}^{4}\left(i^2-4\right)$ Answer: $\sum_{i=-1}^{4}\left(i^2-4\right)=7$

10. Re-write the given summation with the final term in the sum removed

$$\sum_{i=0}^{n} 3^{i+1}$$

Answer: $\sum_{i=0}^{n} 3^{i+1}=\sum_{i=0}^{n-1} 3^{i+1}+3^{n+1}$

11. Find $\sum_{k=0}^{156} k=12,246$

12. Find $\sum_{k=13}^{178} k$

2nd way:

$$\sum_{k=13}^{178} k=\sum_{k=0}^{178} k-\sum_{k=0}^{12} k=\frac{178\times 179}{2}-\frac{12\times 13}{2}=15,931-78=15,853$$

In-class Practice

9. Compute $\sum_{i=-1}^{4}(i^2-4)$ Answer: $\sum_{i=-1}^{4}(i^2-4)=7$

10. Re-write the given summation with the final term in the sum removed

$$\sum_{i=0}^{n}3^{i+1}$$

Answer: $\sum_{i=0}^{n}3^{i+1}=\sum_{i=0}^{n-1}3^{i+1}+3^{n+1}$

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In-class Practice

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$$\sum_{i=0}^{n} 3^{i+1}$$

Answer: $\sum_{i=0}^{n} 3^{i+1} = \sum_{i=0}^{n-1} 3^{i+1} + 3^{n+1}$

11. Find $\sum_{k=0}^{156} k = 12{,}246$

12. Find $\sum_{k=13}^{178} k = 15{,}853$

13. Find $\sum_{k=0}^{6} 3^k = 6\frac{(3^{6+1} - 1)}{(3-1)} = 6{,}558$

$$\sum_{i=0}^{n-1} a_0 r^i = \frac{a_0 (r^n - 1)}{r - 1}$$

In-class Practice

9. Compute $\sum_{i=-1}^{4}(i^2-4)$ Answer: $\sum_{i=-1}^{4}(i^2-4)=7$

10. Re-write the given summation with the final term in the sum removed

$\sum_{i=0}^{n}3^{i+1}$ Answer: $\sum_{i=0}^{n}3^{i+1}=\sum_{i=0}^{n-1}3^{i+1}+3^{n+1}$

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14. Find $\sum_{k=-1}^{20}(3+5k)$

In-class Practice

use $$\sum_{i=1}^{n} a_i = (a_1 + a_n)\frac{n}{2}$$ ← number of terms

first term last term

$$\sum_{k=-1}^{20}(3+5k) = \frac{(-2+103)(22)}{2} = 1{,}111$$

or use $$\sum_{i=1}^{n}(c+di) = cn + d\frac{(1+n)n}{2}$$ ← number of terms

first term last term

$$\sum_{k=-1}^{20}(3+5k) = \sum_{k=-1}^{20} 3 + 5\sum_{k=-1}^{20} k = 3(20+2) + 5\sum_{k=1}^{22}(k-2) =$$

$$= 66 + 5\frac{(-1+20)(22)}{2} = 66 + 1045 = 1{,}111$$

In-class Practice

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$$\sum_{i=0}^{n}3^{i+1}$$

Answer: $\sum_{i=0}^{n}3^{i+1}=\sum_{i=0}^{n-1}3^{i+1}+3^{n+1}$

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