CS135

p. 358/25 give recursive definitions

(a) the set of even integers.

$0 \in S$
$x+2,\ x-2 \in S$ if $x \in S$

checking: $0 \in S$,
$2 \in S$ $(0+2)$
$-2 \in S$ $(0-2)$, ...

(b) the set of positive integers congruent to 2 modulo 3.

$2 \bmod 3 = 2$ $\quad$ $5 \bmod 3 = 2$ $\quad$ $8 \bmod 3 = 2$ ...

$-1 \cancel{\bmod 3} = 2$ $\quad$ $-4 \cancel{\bmod 3 = 2}$ $\quad$ $-7 \cancel{\bmod}\ 3 = 2$ ...

i.e. $2 \equiv 5 \pmod 3$ $\quad$ $8 \equiv 2 \pmod 3$ $\quad$ $2 \equiv \cancel{-1} \pmod 3$

$-4 \cancel{\equiv 2} \pmod 3$ ....

So we need these integers: $2 \xrightarrow{+3} 5 \xrightarrow{+3} 8 \xrightarrow{+3} 11 \xrightarrow{+3} 14, \ldots$

$2 \in S$
$x+3 \in S$ if $x \in S$

(c) the set of positive integers not divisible by 5

$1, 2, 3, 4, 6, 7, \ldots, 9, 11, 12, 13, 14, 16, \ldots 19, 21, \ldots$

$1 \in S$
$x+1 \in S$ if $x \in S$ and $5 \nmid x+1$

5 doesn't divide x+1

or

$1 \in S$
$2 \in S$
$3 \in S$
$4 \in S$
$x+5 \in S$ if $x \in S$