cs135

p. 358/13 prove that $\underbrace{f_1 + f_3 + \dots + f_{2n-1} = f_{2n}}_{P(n)}$, $n \in \mathbb{Z}^+$

Proof (~~structural~~ math induction)

Basis step: when $n = 1$ we get: $f_1 = f_2$

which is true

Inductive step: assume that for an arbitrary fixed $k \geq 1$

$P(k)$ is true, i.e. $f_1 + f_3 + \dots + f_{2k-1} = f_{2k}$ (IH)

Let's show that in this case $P(k+1)$ is also true, i.e.

$$f_1 + f_3 + \dots + f_{2k+1} = f_{2(k+1)} \quad :$$

$$f_1 + f_3 + \dots + f_{2k-1} + f_{2(k+1)-1} = \underbrace{f_1 + f_3 + \dots + f_{2k-1}}_{\text{use } P(k) \text{ for it}} + f_{2k+1} =$$

$$= f_{2k} + f_{2k+1} = f_{2k+2} = f_{2(k+1)}$$

by def. of fibonacci numbers $f_{2k+2} = f_{2k+1} + f_{2k}$

So we proved that $f_1 + f_3 + \dots + f_{2(k)+1} = f_{2(k+1)}$

This completes the inductive step.

By math induction $P(n)$ is true for all $n \in \mathbb{Z}^+$.

q.e.d.