CSI 35 Strong Induction

p. 341/3 $P(n)$: "a postage of n cents can be formed using only 3¢ and 5¢ stamps"

We need to prove that $P(n)$ is true for any $n \in \mathbb{Z}$, $n \geq 8$.

Proof:

a) Basis step:

if $n = 8$¢, then use 3¢ stamp + 5¢ stamp = 8¢

if $n = 9$¢, then use 3¢ stamp + 3¢ stamp + 3¢ stamp = 9¢

if $n = 10$¢, then use 5¢ stamp + 5¢ stamp = 10¢

This concludes basis step.

b) Inductive step: assume for an arbitrary fixed $k \in \mathbb{Z}$, $k \geq 10$ $P(i)$ is true for all $i \in \mathbb{Z}$, $8 \leq i \leq k$ (IH)

c) we need to show that under IH $P(k+1)$ is also true i.e. $(k+1)$¢ postage can be formed using only 3¢ and 5¢ stamps.

d) since $P(k-2)$ is true (checking bound: if $k=10$, then $k-2=8$, by IH $P(8)$ is true - everything is ok)

i.e. $(k-2)$¢ postage can be formed using only 3¢ and 5¢ stamps, so let's add one 3¢ stamp: $(k-2)$¢ + 3¢ = $(k+1)$¢ postage.

- we found the arrangement of stamps for the $(k+1)$¢ postage.

e) By strong induction we proved that $P(n)$ is true for any $n \in \mathbb{Z}$, $n \geq 8$.

q.e.d.