CS135 **Mathematical Induction.**

p. 330/33 — 5 divides $n^5 - n$ if n is non-neg. integer.

Proof:

Basis step: $n = 0$: $5 \mid 0^5 - 0$ indeed.

$\mid$ stands for "divides"

This completes basis step.

Inductive step: assume that $5 \mid k^5 - k$ for any arbitrary fixed integer $k \geq 0$.

We need to prove/show that in this case $5 \mid (k+1)^5 - (k+1)$ also.

$(k+1)^5 - (k+1) =$

recall Pascal's triangle with binomial coefficients

n=1	1
n=2	1 2 1
n=3	1 3 3 1
n=4	1 4 6 4 1
n=5	1 5 10 10 5 1

$(k+1)^1 = k + 1$

$(k+1)^2 = k^2 + 2k + 1$

$(k+1)^3 = k^3 + 3k^2 + 3k + 1$

$(k+1)^4 = k^4 + 4k^3 + 4k^2 + 1$

$(k+1)^5 = k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1$

keep because

I see it there

$$= k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + \cancel{1} - k - \cancel{1} =$$

$$= k^5 - k + 5(k^4 + 2k^3 + 2k^2 + k)$$

$5 \mid k^5 - k$ by I.H.

$5 \mid 5(k^4 + 2k^3 + 2k^2 + k)$

$5 \mid (k^5 - k) + 5(k^4 + 2k^3 + 2k^2 + k)$ by Theorem 1 on page 238

Therefore we showed that $5 \mid (k+1)^5 - (k+1)$

By math. induction $5 \mid n^5 - n$ for any integer $n \geq 0$. q.e.d