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5 divides  $n^5 - n$  if  $n$  is non-neg. integer.

Proof:

Basis step:  $n=0 : 5 \mid 0^5 - 0$  indeed.

$\mid$  stands for "divides"

This completes basis step.

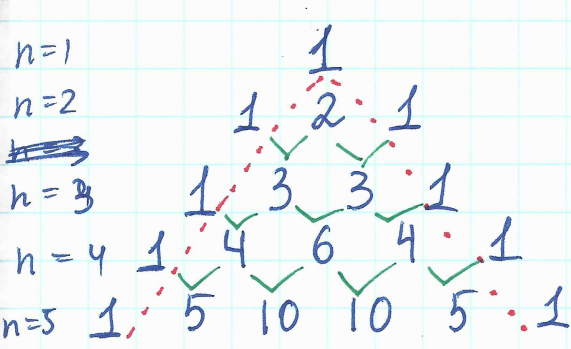
Inductive step:

assume that  $5 \mid k^5 - k$  for any arbitrary fixed integer  $k \geq 0$ .

We need to prove/show that in this case  $5 \mid (k+1)^5 - (k+1)$  also.

$(k+1)^5 - (k+1) =$

recall Pascal's triangle with binomial coefficients



$(k+1)^1 = k+1$   
 $(k+1)^2 = k^2 + 2k + 1$   
 $(k+1)^3 = k^3 + 3k^2 + 3k + 1$   
 $(k+1)^4 = k^4 + 4k^3 + 6k^2 + 4k + 1$   
 $(k+1)^5 = k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1$

I see it there

$k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1$   $-$   $k + 1$   $=$

$k^5 - k$   $+$   $5(k^4 + 2k^3 + 2k^2 + k)$

$5 \mid k^5 - k$   
by IH.

$5 \mid 5(k^4 + 2k^3 + 2k^2 + k)$

$5 \mid (k^5 - k) + 5(k^4 + 2k^3 + 2k^2 + k)$

by Theorem 1 on page 238

Therefore we showed that  $5 \mid (k+1)^5 - (k+1)$   
By math. induction  $5 \mid n^5 - n$  for any integer  $n \geq 0$ .

a.e.d