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$2n + 3 \le 2^n$

for which $n \in \mathbb{Z}^+$?

$n = 1$	$5 \le 2^1$	False	$n = 3$	$9 \le 2^3$	False
$n = 2$	$7 \le 2^2$	False	$n = 4$	$11 \le \underset{16}{2^4}$	True
$n = 5$	$13 \le \underset{32}{2^5}$	True	$n = 6$	$15 \le \underset{64}{2^6}$	True

Answer: it is true for $n \geq 4$.

Proof:

Basis step: $n = 4$: $2 \cdot 4 + 3 \overset{?}{\le} 2^4$

$11 \le 16$ True!

this completes basis step.

Inductive step: assume $\underbrace{2k + 3} \le \underbrace{2^k}$ for any arbitrary fixed integer $k \geq 4$. (IH)

we need to show that in this case:

$\boxed{2(k+1) + 3} \le 2^{k+1}$ also

$\boxed{2(k+1) + 3} = 2k + 5 = \underbrace{2k+3} + 2 \le \underbrace{2^k}_{\text{(by IH)}} + 2 \le$

$2 < 2^k$ for $k \geq 4$

$2^k + 2^k = 2^{k+1}$

this completes inductive step.

By math. induction we proved that

$2n + 3 \le 2^n$ for any integer $n \geq 4$

q.e.d.