p. 329/5

$\forall n \in \mathbb{W}\ P(n)$

Prove that

$$1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

for $\forall n \in \mathbb{Z}$ and $n \geq 0$ i.e. for $\forall n \in \mathbb{W}$

whole numbers.

<u>Proof:</u>

<u>Base step:</u> $n = 0$ $\quad 1^2 = 1$ equal

$$\frac{(n+1)(2n+1)(2n+3)}{3} = \frac{(0+1)(0+1)(0+3)}{3} = 1$$

hence $P(0)$ is true. This completes base step.

<u>Inductive step:</u> assume that for any arbitrary fixed

(IH) $k \in \mathbb{W}$, $P(k)$ is true, i.e. $1^2 + \dots + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$

<u>we need to show:</u> $1^2 + 3^2 + \dots + (2k+1)^2 + (2(k+1)+1)^2 =$

$$= \frac{(k+2)(2k+3)(2k+5)}{3}$$

I see it there

let's show it:

$$P(k+1) = 1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 + (2(k+1)+1)^2 = \underbrace{1^2 + 3^2 + 5^2 + \dots + (2k+1)^2}_{P(k)} + (2k+3)^2 =$$

$$\underset{\text{(by IH)}}{=} \underbrace{\frac{(k+1)(2k+1)(2k+3)}{3}} + (2k+3)^2 = \frac{(k+1)(2k+1)(2k+3) + 3(2k+3)^2}{3} =$$

$$\underset{\substack{\text{factor out}\\ 2k+3 \\ \text{because}}}{=} \frac{(2k+3)\big((k+1)(2k+1) + 3(2k+3)\big)}{3} = \frac{(2k+3)(2k^2+3k+1+6k+9)}{3} =$$

$$= \frac{(2k+3)(2k+9k+10)}{3} = \frac{(2k+3)(k+2)(2k+5)}{3}$$

$(2k+9k+10) = (k+2)(2k+5)$

hence $P(k+1)$ is true.

By math. induction the formula is correct for all $n \in \mathbb{W}$. q.e.d.