CSI 35 Summations and Sequences <u>answers and solutions</u>

① $\boxed{-7}, 8, 5, -3, 12, 5, \boxed{27}$

↑initial term ↑final term

② $b_i = (-1)^i \dfrac{2^i}{2i}$ for $i \geq 3$

↑ so we will start with $i = 3$ (initial index $= 3$)

<u>first four terms:</u>

1) $i = 3$ $b_3 = (-1)^3 \dfrac{2^3}{2 \cdot 3} = -\dfrac{2^3}{6} = -\dfrac{2^2}{3} = -\dfrac{4}{3}$

$b_3 = -\dfrac{4}{3}$

2) $i = 4$ $b_4 = (-1)^4 \dfrac{2^4}{2 \cdot 4} = (-1)^4 \dfrac{2^4}{2^3} = 2$

$b_4 = 2$

3) $i = 5$ $b_5 = (-1)^5 \dfrac{2^5}{2 \cdot 5} = -\dfrac{2^4}{5} = -\dfrac{16}{5}$ $b_5 = -\dfrac{16}{5}$

4) $i = 6$ $b_6 = (-1)^6 \dfrac{2^6}{2 \cdot 6} = \dfrac{2^6}{2^2 \cdot 3} = \dfrac{2^4}{3} = \dfrac{16}{3}$ $b_6 = \dfrac{16}{3}$

Answer: $b_3 = -\dfrac{4}{3}$, $b_4 = 2$, $b_5 = -\dfrac{16}{5}$, $b_6 = \dfrac{16}{3}$

or simply

$-\dfrac{4}{3}, 2, -\dfrac{16}{5}, \dfrac{16}{3}$

③ 3, 8, 13, 18, 23, 28, 33 ← a_7

↑a_1 ↑a_3

a) $a_5 = 23$

b) 3

c) 7

④ arithmetic series (sequence)

② , 9, 16, 23, 30, … ← means "infinite".

a) ↑ initial value

b) $9-2=7$, $16-9=7$, …. hence $\boxed{\text{common difference} = 7}$

c) $a_0 = 2$, $d = 7$, use formula $a_n = a_0 + dn$

$$a_{25} = 2 + 7 \cdot 25 = 177 \qquad \boxed{S_{25} = 177}$$

⑤ geometric sequence (progression, …) $\{s_n\}$

$$\overset{\uparrow}{\underset{s_0}{-1}},\ \overset{s_1}{7},\ \overset{s_2}{-49},\ \overset{s_3}{343},\ \ldots.$$

a) -1

b) $7 \div (-1) = \underline{\underline{-7}}$, $(-49) \div 7 = \underline{\underline{-7}}$, … hence

$\boxed{\text{common ratio} = -7}$

c) $s_4 = 343 \cdot (-7) = -2401$ or $s_4 = (-1) \cdot (-7)^4 = -2401$

$\boxed{s_4 = -2401}$

d) $s_{32} = (-1)(-7)^{32} = -7^{32}$ using formula

$$s_n = s_0 \cdot r^n$$

$\boxed{s_{32} = -7^{32}}$

⑥ $\displaystyle\sum_{j=0}^{3} (j+3)^2 = 9 + 16 + 25 + 36 = 86$

$$\boxed{\sum_{j=0}^{3} (j+3)^2 = 86}$$

when $j = 0$, $(j+3)^2 = (0+3)^2 = 3^2 = 9$

when $j = 1$, $(j+3)^2 = (1+3)^2 = 4^2 = 16$

when $j = 2$, $(j+3)^2 = (2+3)^2 = 5^2 = 25$

when $j = 3$ $(j+3)^2 = (3+3)^2 = 6^2 = 36$

⑦ $$\sum_{i=-3}^{2}(2i+5) = \underset{i=-3}{(2\cdot(-3)+5)} + \underset{i=-2}{(2\cdot(-2)+5)} +$$

$$\underset{i=-1}{(2\cdot(-1)+5)} + \underset{i=0}{(2\cdot 0+5)} + \underset{i=1}{(2\cdot 1+5)} + \underset{i=2}{(2\cdot 2+5)} =$$

$$= \cancel{(-1)} + \cancel{(1)} + 3 + 5 + 7 + 9 = 24$$

$$\sum_{i=-3}^{2}(2i+5) = 24$$

⑧ $$\sum_{i=0}^{1000}(-1)^{i} = \boxed{\sum_{i=0}^{999}(-1)^{i} + 1}$$

when $i=1000$, $(-1)^{1000} = 1$ ← last term

⑨ $$\sum_{j=-3}^{k+4} 3^{j-3} = \boxed{\sum_{j=-3}^{k+3} 3^{j-3} + 3^{k+1}}$$

when $j = k+4$, $3^{j-3} = 3^{(k+4)-3} = 3^{k+1}$ ← last term

⑩ $$\sum_{i=-5}^{30} i^{4} = \sum_{i=1}^{?} ? = \sum_{i=1}^{30+6}(i-6)^{4}$$

↑ to make it 1 we need to add 6, hence add 6 to both limits and subtract 6 from i in the expression

$$\boxed{\sum_{i=-5}^{30} i^{4} = \sum_{i=1}^{36}(i-6)^{4}}$$

⑪ $$\sum_{j=-1}^{n}\frac{3}{j+7} = \sum_{i=2}^{?} ? = \sum_{i=2}^{n+3}\frac{3}{(i-3)+7} = \sum_{i=2}^{n+3}\frac{3}{i+4}$$

change j to i, and add 3, and subtract 3 from i in the expression

$$\boxed{\sum_{j=1}^{n}\frac{3}{j+7} = \sum_{i=2}^{n+3}\frac{3}{i+4}}$$

⑬ $$\sum_{i=0}^{5}(i+1) = \underset{i=0}{1} + \underset{i=1}{2} + \underset{i=2}{3} + \underset{i=3}{4} + \underset{i=4}{5} + \underset{i=5}{6} = \boxed{21}$$

1)

$\boxed{\text{or}}$ use formula $$\sum_{i=0}^{\boxed{n-1}}(\underset{1}{a_0} + \underset{1}{d}\,\underset{i}{i}) = \underset{1}{a_0}\cdot \underset{6}{n} + \underset{1}{d}\,\frac{\overset{6}{n}(\overset{6}{n-1})}{2}$$

2) $$\sum_{i=0}^{5}(i+1)$$

partial sum of arithm. sequence

matching

5

$$\sum_{i=0}^{5}(i+1) = 1\cdot 6 + 1\cdot\frac{6(6-1)}{2} = 6 + 3\cdot 5 = 6 + 15 = \boxed{21}$$

$\boxed{\text{or}}$ ~~use~~ formula $$\sum_{i=0}^{n-1} a_i = \frac{(a_0 + a_{n-1})\,n}{2}$$

3) $$\sum_{i=0}^{5}(i+1)$$

$a_0 = 1$ (when $i=0$)

$a_{n-1} = 6$ (when $i=5$)

$n = 6$

$$= \frac{(1+6)\,6}{2} = \boxed{21}$$

$$\boxed{\text{Answer: } \sum_{i=0}^{5}(i+1) = 21}$$

⑭ Find $\underset{a_0}{3} + 6 + 9 + 12 + 15 + \ldots + 597$ ← let's find its index

$d = 3$ (common difference)

$6-3=3,\ 9-6=3,\ 12-9=3$

use $a_n = a_0 + d\cdot n$

$597 = 3 + 3\cdot n$ $\quad n = 198$ $\quad$ so $a_{198} = 597$

use $$\sum_{i=0}^{n-1} a_0 = \frac{(a_0 + a_{n-1})\,n}{2}$$

$$\sum_{i=0}^{198} a_0 = \frac{(\overset{\text{first}}{3} + \overset{\text{last}}{597})\,(\overset{\text{number of terms}}{198+1})}{2} = 59700$$

$\boxed{\text{Answer: } 59700}$

⑮ Find $\underset{\uparrow}{3} + 6 + 12 + 24 + \dots + \underset{\uparrow}{768}$ geometric progression (sequence)

$a_0 = 3$ common ratio, $r = 2$ $\left(\frac{6}{3} = 2, \frac{12}{6} = 2, \dots\right)$

what is its index ?

use formula $a_n = a_0 \cdot r^n$

$768 = 3 \cdot 2^n$ $256 = 2^n$

$256 = 2^8$, so $n = 8$

$$\sum_{i=0}^{8} 3 \cdot 2^i = \frac{3(2^9 - 1)}{2 - 1} = 3(511) = 1533$$

- used formula

$$\sum_{i=0}^{n-1} a_0 \cdot r^i = \frac{a_0(r^n - 1)}{r - 1}$$

$$\boxed{\text{Answer: } 1533}$$

⑯ $$\sum_{i=0}^{237} (2i + 11) = 2\sum_{i=0}^{237} i + 11(237 + 1) = \cancel{2} \cdot \frac{237 \cdot 238}{\cancel{2}} + 11 \cdot 238 =$$

used formulas: $\sum_{i=1}^{k} c = kc$ and $\sum_{i=0}^{n-1} (c + di) = \sum_{i=0} c + d\sum_{i=0} i$

$$\sum_{i=0}^{n-1} i = \frac{(n-1)n}{2}$$

$= 237 \cdot 238 + 11 \cdot 238 = 248 \cdot 238 = 59024$ $\boxed{\sum_{i=0}^{237} (2i + 11) = 59024}$

⑰ $$\sum_{i=-5 \; +6}^{28 \; +6} 13 = \sum_{i=1}^{34} 13 = 13 \cdot 34 = 442$$

change lower limit to 1

used $\sum_{i=1}^{K} c = Kc$

$$\boxed{\sum_{i=-5}^{28} 13 = 442}$$