CSI 35 Chapter 5 review

1. Compute $$\sum_{i=-2}^{26} (5i-8)$$

2. Rewrite $$\sum_{j=-3}^{n+1} (j+5)^3$$

so that the lower limit is 1

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3. Find the sum $15 + 19 + 23 + 27 + \ldots + 403$

4. Find the sum $3 + (-12) + 48 + \ldots + 12{,}288$

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5. Is the following proof correct? Justify your answer.

Let's prove that $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(n-1)n}=\frac{3}{2}-\frac{1}{n}$ if $n \in \mathbf{Z}^+$

let *P(n)* stand for

Basis step: The result is true when *n=1* because $\frac{1}{1\cdot 2}=\frac{3}{2}-\frac{1}{1}$

Inductive step: assume that the result, *P(k)*, is true for an integer value $k \geq 1$ (IH), i.e. $\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(k-1)k}=\frac{3}{2}-\frac{1}{k}$

Let's show that in this case *P(k+1)* is also true:

$$\frac{1}{1\cdot 2}+\frac{2}{1\cdot 3}+\ldots+\frac{1}{(k-1)k}+\frac{1}{((k+1)-1)(k+1)}=by\ IH=\frac{3}{2}-\frac{1}{k}+\frac{1}{((k+1)-1)(k+1)}=$$

$$=\frac{3}{2}-\frac{1}{k}+\frac{1}{k(k+1)}=\frac{3}{2}+\frac{-(k+1)+1}{k(k+1)}=\frac{3}{2}+\frac{-k}{k(k+1)}=\frac{3}{2}-\frac{1}{k+1}$$

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…

We proved that *P(k+1)* is also true. This completes the inductive step. By mathematical induction we proved that *P(n)* is true for all positive integers.

q.e.d.

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6. Use *mathematical induction* to prove that for any positive integer *n*

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

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7. Use *mathematical induction* to show that $2^n > n^2 + n$ for all integer values $n > 4$.

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8. Use *mathematical induction* to prove that 3 divides n^3+2n for any positive integer n.

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9. Give a recursive definition of
a) the set of pairs of positive integers whose sum is odd
b) the set of bit strings
c) the set of rooted, binary trees

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10. Merge sort the list *8,2,4,5,10,9,7,3,1* in *increasing order*.