CSI 35 **Chapter 12 review** **ZyBooks**

1. Let S be the set of all strings of English letters. Determine whether the following relations are *reflexive*, *anti-reflexive*, *symmetric*, *anti-symmetric*, and/or *transitive*.

a) R = { (*a,b*) | *a* and *b* have no letters in common}

b) S = { (*a,b*) | *a* and *b* are not the same length}

c) T = { (*a,b*) | *a* is longer than *b*}

2. Is relation R on $\mathbf{Z} \times \mathbf{Z}$ defined by (a,b) R (c,d) iff a+d = b+c an equivalence relation?

3. The relation R on the set of all integers is defined as:
$(x,y) \in R$ iff $xy \geq 0$. Determine whether the relations is *reflexive*, *anti-reflexive*, *symmetric*, *anti-symmetric*, and/or *transitive*.

4. Let R be relation on the set {1,2,3,4,5} containing the ordered pairs (1,1), (1,2), (1,3), (2,3), (2,4), (3,1), (3,4), (3,5), (4,1), (4,5), (5,1), (5,2), and (5,4).

a) Give its *graph representation* (*arrow diagram*)

b) Give its *matrix representation*

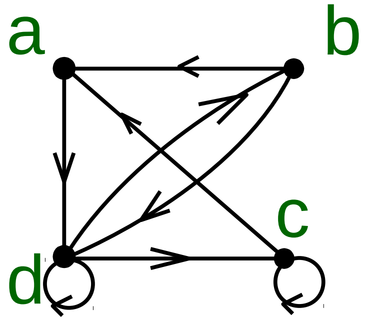
c) Find R^2

d) Find R^3

5. Let R_1 and R_2 be relations on the set {a,b,c,d}.
R_1 = {(a,b), (b,c), (b,d), (d,a), (d,b)}
R_2 = {(a,c), (a,d), (b,c), (b,d), (d,a), (d,c), (d,d)}

Find a) $R_1 \circ R_2$ b) $R_2 \circ R_1$

6. List the ordered pairs in the relation represented by the graph

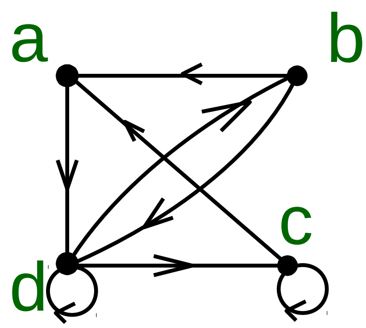


7. List the ordered pairs in the relation on {1,2,3,4} represented by the matrix
(the integers are listed in increasing order)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

8. Relation R is represented by the matrix M and relation S by the directed graph G. Both relations are on the set of {a,b,c,d}. Determine whether the relations are *reflexive*, *anti-reflexive*, *symmetric*, *anti-symmetric*, and/or *transitive*.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$



9. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad M_{R2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrix that represents $R_1 \circ R_2$

10. Which of these are posets? If a poset, is it a *total order*?

a) (**R**, =) b) (**R**, <) c) (**R**, $\leq$) d) (**R**, $\neq$)

(i.e. *reflexive*, *anti-symmetric*, and *transitive*.)

11. Consider the poset ({2,4,6,9,12,18,27,36,48,60,72}, |).

a) Draw Hasse diagram of the poset
b) Find the maximal elements.
c) Find the minimal elements.

12. Is it possible to have a relation on the set {a, b, c} that is both reflexive and anti-reflexive?
If so, give an example.

13. Is it possible to have a relation on the set {a, b, c} that is both symmetric and anti-symmetric? If so, give an example.

14. For the relation $R = \{(1,1), (1,b), (1,c), (c,b), (c,c)\}$ on the set $S = \{1,b,c\}$, find the transitive closure R^+ of R. Feel free to use directed graphs or matrix representations, but provide the answer as a set of pairs.

15. Are the following relations partial orders?

(a) The domain is a group of brothers and sisters.
$x \preceq y$ if y is at least as old as x.
You can assume that all the brothers and sisters have the same mother, so no two of them were born at exactly the same time.

(b) The domain is the set of people working at a company.
$x \preceq y$ if y has a higher salary than x.

16. For the following relation, indicate whether the relation is a partial order, a strict order, or neither. If the relation is a partial or strict order, indicate whether the relation is also a total order.

The domain is the set of all words in the English language (as defined by, say, Webster's dictionary).
Word x is related to word y if x appears before y in alphabetical order.
Assume that each word appears exactly once in the dictionary.