CSI 35 **Chapter 11 review** **ZyBooks**

1. Find the sum *-15 + -11 + -7 + … + 405*

2. Find the sum *3 + (-12) + 48 + … + 12,288* (geometric sequence)

3. Given an *arithmetic sequence* with *initial term* 19, and the *common difference* of 3, find the term with index 35, assuming that the *arithmetic sequence* starts with an *initial index* of 0.

4. Given a geometric sequence with initial term of $-\frac{5}{16}$ and the common ratio -2, find the term with index 5, assuming that the *sequence* starts with an *initial index* of 0.

5. Compute $\sum_{j=-5}^{27}(5-2j)$

6. Rewrite $\sum_{k=8}^{n+5}(2k^2-7k)$ so that the lower limit is 0.

7. Use *mathematical induction* to prove that for any positive integer n,

$$1\cdot2\cdot3+2\cdot3\cdot4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

8. Use *mathematical induction* to prove that $2^n < n!$ For every integer n with $n \geq 4$.

9. Use *mathematical induction* to prove that 3 divides n^3+2n for any positive integer n.

10. Use *mathematical induction* to show that if S is a finite set with n elements, where n is a non-negative integer, then S has 2^n subsets.

12. Find a formula for $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}$ by examining the values of this expression for small values of n. Prove the formula your found.

13. What is wrong with this "proof" that all horses are the same color?

Let *P(n)* be the proposition that all the horses in a set of *n* horses are the same color.

Basis step: Clearly, P(1) is true.

Inductive step: Assume *P(k)* is true, so that all the horses in any set of *k* horses are the same color.
Consider any *(k+1)* horses; number these as horses *1, 2, 3, 4, …,k, k+1*.
Now the first *k* of these horses all must have the same color, and the last *k* of these horses must also have the same color. Because the set of the first *k* horses and the last *k* horses overlap, all *(k+1)* horses must be the same color.
This shows that *P(k+1)* is true and finishes the proof by induction!

14. Find `f(2)`, `f(3)`, `f(4)`, and `f(5)` if f is defined recursively by

```
f(0) = -1,
f(1) = 2, and
f(n) = 3f(n-1)²-4f(n-2)², for n = 2, 3, ….
```

15. Determine whether the proposed recursive definition is a valid recursive definition:

```
f(0) = 1
f(n) = f(n-1)  if n is odd and n ≥ 1
f(n) = 9f(n-2) if n is even and n ≥ 1
```

16. Give a recursive definition of the sequence $\{a_n\}$, n = 1, 2, 3, 4… if $a_n=10^n$

17. Give a recursive definition of
a) the set of pairs of positive integers whose sum is odd
b) the set of bit strings
c) the set of rooted, binary trees
d) the set of rooted, full binary trees

18. Let S be the subset of ordered pairs of integers defined recursively by

Basis step: $(0,0) \in S$
Recursive step: if $(a,b) \in S$, then $(a,b+1), (a+1,b+1),(a+2,b+1) \in S$

List the elements of S produced by the first four applications of recursive definition.

19. Give a recursive definition of the functions *ones(s)*, which counts the number of ones in a bit string *s*.

20. Give a recursive algorithm for finding the sum of the first *n* odd positive integers.

21. Write a recursive algorithm for calculating the sum in *problem 5*.

Additional practice:

1. Use *mathematical induction* to prove that the sum of the first n positive odd integers is n^2, that is, $1 + 3 + 5 + \ldots. + (2n-1) = n^2$.

2. Use *mathematical induction* to show that for all non-negative integers n,

$$1+2+2^2+\ldots+2^n=2^{n+1}-1$$

3. Use *mathematical induction* to prove inequality $n < 2^n$ for all positive integers n.

4. Use mathematical induction to prove that $2^n < n!$ For every integer n with $n \geq 4$.

5. Use mathematical induction to prove that 5 divides n^5-n whenever n is a non-negative integer.