Chapter 9 Formulas / Theorems sheet

Properties of relations:

A relation R on set A is *reflexive* if $(a,a) \notin R$ for every $a \in A$.

A relation R on set A is *irreflexive* if $(a,a) \in R$ for every $a \in A$.

A relation R on set A is *symmetric* if $(b,a) \in R$ whenever $(a,b) \in R$, for all $a, b \in A$.

A relation R on set A is *antisymmetric* if whenever $(b,a) \in R$ and $(a,b) \in R$, then a=b, for all $a, b \in A$.

A relation R on set A is *transitive* if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$, for all $a, b \in A$.

A relation R on set A is *asymmetric* if $(a,b) \in R$ implies that $(b,a) \notin R$.

A relation R on set A is *antisymmetric* if for all $a, b \in A$ if $(b,a) \in R$ and $(a,b) \in R$, then $a = b$.

Some correlations:

If a relation is reflexive, then it is not asymmetric.

Irreflexive is not opposite of reflexive.

Asymmetric is not opposite of symmetric.

Antisymmetric is not opposite of symmetric.

Let R be a relation on set **A**. The powers R^n , $n=1,2,3,...$ are defined recursively by $R^1 = R$

$$R^{n+1} = R^n \circ R$$

A relation on a set A is called an *equivalence relation* if it is *reflexive, symmetric* and *transitive*.

Let R be an equivalence relation on a set A. The set of all elements that are related to an element a of A is called the *equivalence class* of a. denotation: $[a]_R$ equivalence class of a , or $[a]$

If $b \in [a]_R$, then b is called *representative* of this equivalence class.

A relation R on a set S is called a *partial ordering* or *partial order* if it is *reflexive, antisymmetric*, and *transitive*. (S,R) is called a *partially ordered set* or *poset*.

The elements a and b of the poset (S,R) are called *comparable* if either aRb or bRa. Otherwise they are called *incomparable*.

If (S,R) is a poset, and every two elements of S are comparable, then S is called a *totally ordered* or *linearly ordered set*, and R is called a *total order* or a *linear order*.
A totally ordered set is called a *chain*.

a is maximal in the poset (S, R) if there is no $b \in S$ such that a R b.

a is minimal in the poset (S, R) if there is no $b \in S$ such that b R a.

a is the greatest element of the poset (S, R) if b R a for all $b \in S$. The greatest element is unique if it exists.

a is the least element of the poset (S, R) if a R b for all $b \in S$. The least element is unique if it exists.

If u is an element of S such that a R u for all elements $a \in A$, then u is called the *upper bound* of A.

If l is an element of S such that l R a for all elements $a \in A$, then l is called the *lower bound* of A.