In practice, algorithms have been developed that can solve traveling salesperson problems with as many as 1000 vertices within 2% of an exact solution using only a few minutes of computer time. For more information about the traveling salesperson problem, including history, applications, and algorithms, see the chapter on this topic in *Applications of Discrete Mathematics* [MiRo91] also available on the website for this book.

Exercises

1. For each of these problems about a subway system, describe a weighted graph model that can be used to solve the problem.

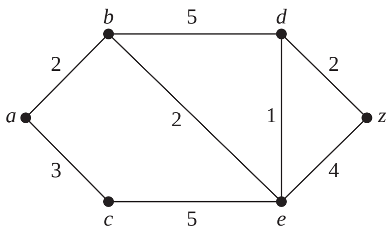
a) What is the least amount of time required to travel between two stops?

b) What is the minimum distance that can be traveled to reach a stop from another stop?

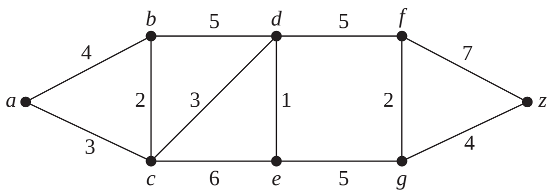
c) What is the least fare required to travel between two stops if fares between stops are added to give the total fare?

In Exercises 2–4 find the length of a shortest path between a and z in the given weighted graph.

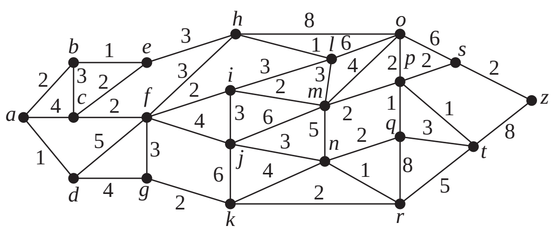
2.



3.



4.



5. Find a shortest path between a and z in each of the weighted graphs in Exercises 2–4.

6. Find the length of a shortest path between these pairs of vertices in the weighted graph in Exercise 3.

a) a and d

b) a and f

c) c and f

d) b and z

7. Find shortest paths in the weighted graph in Exercise 3 between the pairs of vertices in Exercise 6.

8. Find a shortest path (in mileage) between each of the following pairs of cities in the airline system shown in Figure 1.

a) New York and Los Angeles

b) Boston and San Francisco

c) Miami and Denver

d) Miami and Los Angeles

9. Find a combination of flights with the least total air time between the pairs of cities in Exercise 8, using the flight times shown in Figure 1.

10. Find a least expensive combination of flights connecting the pairs of cities in Exercise 8, using the fares shown in Figure 1.

11. Find a shortest route (in distance) between computer centers in each of these pairs of cities in the communications network shown in Figure 2.

a) Boston and Los Angeles

b) New York and San Francisco

c) Dallas and San Francisco

d) Denver and New York

12. Find a route with the shortest response time between the pairs of computer centers in Exercise 11 using the response times given in Figure 2.

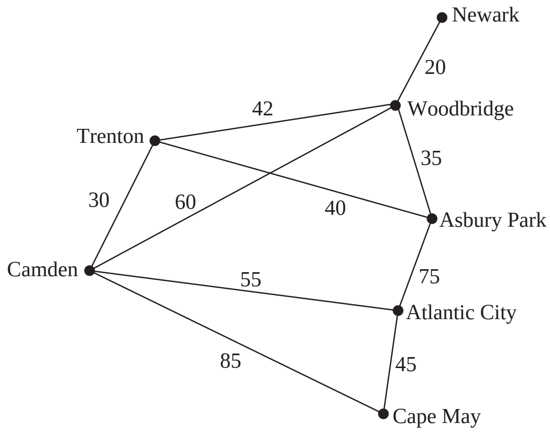
13. Find a least expensive route, in monthly lease charges, between the pairs of computer centers in Exercise 11 using the lease charges given in Figure 2.

14. Explain how to find a path with the least number of edges between two vertices in an undirected graph by considering it as a shortest path problem in a weighted graph.

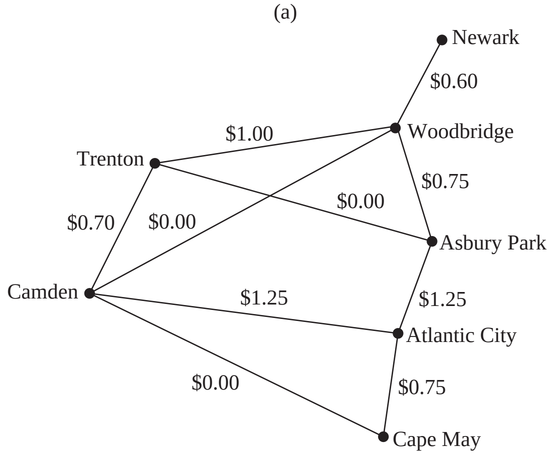
15. Extend Dijkstra's algorithm for finding the length of a shortest path between two vertices in a weighted simple connected graph so that the length of a shortest path between the vertex a and every other vertex of the graph is found.

16. Extend Dijkstra's algorithm for finding the length of a shortest path between two vertices in a weighted simple connected graph so that a shortest path between these vertices is constructed.

17. The weighted graphs in the figures here show some major roads in New Jersey. Part (a) shows the distances between cities on these roads; part (b) shows the tolls.



(a)



(b)

a) Find a shortest route in distance between Newark and Camden, and between Newark and Cape May, using these roads.

b) Find a least expensive route in terms of total tolls using the roads in the graph between the pairs of cities in part (a) of this exercise.

18. Is a shortest path between two vertices in a weighted graph unique if the weights of edges are distinct?

19. What are some applications where it is necessary to find the length of a longest simple path between two vertices in a weighted graph?

20. What is the length of a longest simple path in the weighted graph in Figure 4 between a and z? Between c and z?

Floyd's algorithm, displayed as Algorithm 2, can be used to find the length of a shortest path between all pairs of vertices in a weighted connected simple graph. However, this algorithm cannot be used to construct shortest paths. (We assign an infinite weight to any pair of vertices not connected by an edge in the graph.)

21. Use Floyd's algorithm to find the distance between all pairs of vertices in the weighted graph in Figure 4(a).

***22.** Prove that Floyd's algorithm determines the shortest distance between all pairs of vertices in a weighted simple graph.

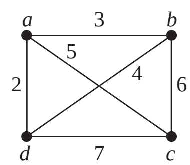
***23.** Give a big-O estimate of the number of operations (comparisons and additions) used by Floyd's algorithm to determine the shortest distance between every pair of vertices in a weighted simple graph with n vertices.

***24.** Show that Dijkstra's algorithm may not work if edges can have negative weights.

ALGORITHM 2 Floyd's Algorithm.

procedure *Floyd*(G: weighted simple graph)
{G has vertices $v_1, v_2, \ldots, v_n$ and weights $w(v_i, v_j)$
 with $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge}
for $i := 1$ **to** n
  for $j := 1$ **to** n
    $d(v_i, v_j) := w(v_i, v_j)$
for $i := 1$ **to** n
  for $j := 1$ **to** n
    for $k := 1$ **to** n
      if $d(v_j, v_i) + d(v_i, v_k) < d(v_j, v_k)$
        then $d(v_j, v_k) := d(v_j, v_i) + d(v_i, v_k)$
return $[d(v_i, v_j)]$ {$d(v_i, v_j)$ is the length of a shortest path between v_i and v_j for $1 \le i \le n, 1 \le j \le n$}

25. Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.



26. Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.