

DEFINITION 2

Let n be a nonnegative integer and G a directed graph. A *path* of length n from u to v in G is a sequence of edges e_1, e_2, \dots, e_n of G such that e_1 is associated with (x_0, x_1) , e_2 is associated with (x_1, x_2) , and so on, with e_n associated with (x_{n-1}, x_n) , where $x_0 = u$ and $x_n = v$. When there are no multiple edges in the directed graph, this path is denoted by its vertex sequence $x_0, x_1, x_2, \dots, x_n$. A path of length greater than zero that begins and ends at the same vertex is called a *circuit* or *cycle*. A path or circuit is called *simple* if it does not contain the same edge more than once.

Remark: Terminology other than that given in Definition 2 is often used for the concepts defined there. In particular, the alternative terminology that uses *walk*, *closed walk*, *trail*, and *path* (described in the remarks following Definition 1) may be used for directed graphs. See [GrYe05] for details.

Note that the terminal vertex of an edge in a path is the initial vertex of the next edge in the path. When it is not necessary to distinguish between multiple edges, we will denote a path e_1, e_2, \dots, e_n , where e_i is associated with (x_{i-1}, x_i) for $i = 1, 2, \dots, n$, by its vertex sequence x_0, x_1, \dots, x_n . The notation identifies a path only as far as which the vertices it passes through. There may be more than one path that passes through this sequence of vertices, which will happen if and only if there are multiple edges between two successive vertices in the list.

Paths represent useful information in many graph models, as Examples 2–4 demonstrate.

EXAMPLE 2

Paths in Acquaintanceship Graphs In an acquaintanceship graph there is a path between two people if there is a chain of people linking these people, where two people adjacent in the chain know one another. For example, in Figure 6 in Section 10.1, there is a chain of six people linking Kamini and Ching. Many social scientists have conjectured that almost every pair of people in the world are linked by a small chain of people, perhaps containing just five or fewer people. This would mean that almost every pair of vertices in the acquaintanceship graph containing all people in the world is linked by a path of length not exceeding four. The play *Six Degrees of Separation* by John Guare is based on this notion. ◀

EXAMPLE 3

Paths in Collaboration Graphs In a collaboration graph, two people a and b are connected by a path when there is a sequence of people starting with a and ending with b such that the endpoints of each edge in the path are people who have collaborated. We will consider two particular collaboration graphs here. First, in the academic collaboration graph of people who have written papers in mathematics, the **Erdős number** of a person m (defined in terms of relations in Supplementary Exercise 14 in Chapter 9) is the length of the shortest path between m and the extremely prolific mathematician Paul Erdős (who died in 1996). That is, the Erdős number of a mathematician is the length of the shortest chain of mathematicians that begins with Paul Erdős and ends with this mathematician, where each adjacent pair of mathematicians have written a joint paper. The number of mathematicians with each Erdős number as of early 2006, according to the Erdős Number Project, is shown in Table 1.

In the Hollywood graph (see Example 3 in Section 10.1) two actors a and b are linked when there is a chain of actors linking a and b , where every two actors adjacent in the chain have acted in the same movie. In the Hollywood graph, the **Bacon number** of an actor c is defined to be the length of the shortest path connecting c and the well-known actor Kevin Bacon. As new movies are made, including new ones with Kevin Bacon, the Bacon number of actors can change. In Table 2 we show the number of actors with each Bacon number as of early 2011 using data from the Oracle of Bacon website. The origins of the Bacon number of an actor dates back to the early 1990s, when Kevin Bacon remarked that he had worked with everyone in Hollywood or someone who worked with them. This led some people to invent a party

Replace Kevin Bacon by your own favorite actor to invent a new party game