CSI33 Data Structures

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Bronx Community College

- Sorting
- The Tower of Hanoi
- Divide-and-conquer Approach
- In-Class Work

Selection Sort

Selection Sort

Recall Selection Sort you were asked to program in one of the hws:

```
def SelectionSort(lst):
   n = len(lst)
   for i in range(n-1):
      pos = i
      for j in range(i+1, n):
         if lst[j] < lst[pos]:
            pos = j
      lst[i], lst[pos] = lst[pos], lst[i]
```

Selection Sort

Selection Sort Analysis

- Inner loop runs n times
- First time it compares n items, then $n-1$, etc.
- Total comparisons $= n + (n-1) + (n-2) + \ldots + 1 = \frac{n(n+1)}{2}$
- Running time is $\Theta(n^2)$

Recursive Design: Mergesort

Mergesort Pseudocode

Now, let's take a look at a recursive sorting algorithm:

```
Algorithm:  mergeSort nums

   split nums into two halves (nums1, nums2)
   sort nums1 (the first half)
   sort nums2 (the second half)
   merge nums1 and nums2 back into nums
```

Recursive Design: Mergesort

Merge Pseudocode

```
Algorithm: merge sorted lists (nums1 and nums2) into nums:
   while both nums1 and nums2 have more items:
      if top of nums1 is smaller:
         copy it into current spot in nums
      else (top of nums2 is smaller):
         copy it into current spot in nums
   copy remaining items from nums1 or nums2 to nums
```

See the definition of the `merge` function in `mergeSort.py`.

Recursive Design: Mergesort

Recursive mergeSort - with the base case

```
if len(nums) > 1:
   split nums into two halves (nums1, nums2)
   mergeSort nums1 (the first half)
   mergeSort nums2 (the second half)
   merge nums1 and nums2 back into nums
```

See the definition of the `mergeSort` function in `mergeSort.py`.

Analyzing Mergesort

Running Time of merge

- Each item gets moved exactly once back into `nums`
- Running time is $\Theta(n)$, where n is the size of `nums`

Merge Pseudocode

```
Algorithm:  merge sorted lists (nums1 and nums2) into nums:
   while both nums1 and nums2 have more items:
      if top of nums1 is smaller:
         copy it into current spot in nums
      else (top of nums2 is smaller):
         copy it into current spot in nums
   copy remaining items from nums1 or nums2 to nums
```

Analyzing Mergesort

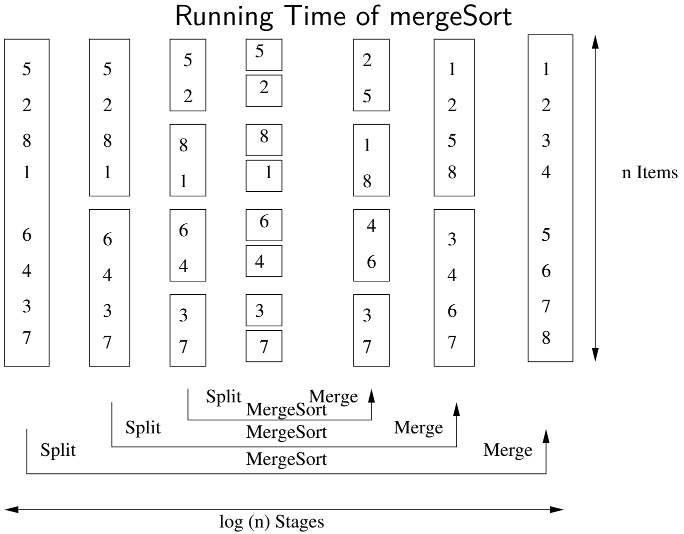
Running Time of mergeSort

- The call stack gets as deep as $\log_2(n)$, where n is the size of `nums`
- At each stage, `mergeSort` is called twice, but for each call, the argument list is half the size as before.
- For $\log_2(n)$ stages, each of the n items is moved once per stage.
- The running time is the product, which is $\Theta(n \log n)$

Recursive mergeSort - with the base case

```
if len(nums) > 1:
   split nums into two halves (nums1, nums2)
   mergeSort nums1 (the first half)
   mergeSort nums2 (the second half)
   merge nums1 and nums2 back into nums
```

Analyzing Mergesort



Tower of Hanoi rules

Tower of Hanoi or Tower of Brahma is a puzzle generally attributed to the French mathematician Édouard Lucas, who published an article about it in 1883.

Read the legend surrounding the puzzle on page 207 in the book.

Tower of Hanoi rules

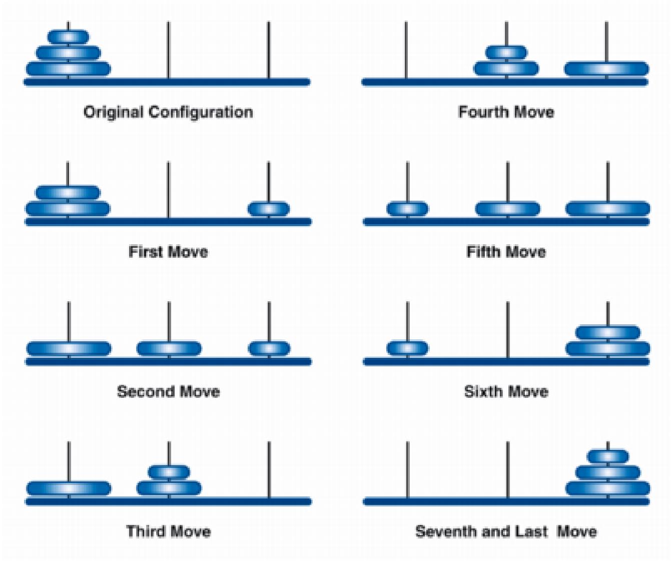
The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack.
- No disk may be placed on top of a smaller disk.

With three disks, the puzzle can be solved in seven moves. The minimum number of moves required to solve a Tower of Hanoi puzzle is $2^n - 1$, where n is the number of disks.

Tower of Hanoi rules

3-disks Tower of Hanoi with solution:

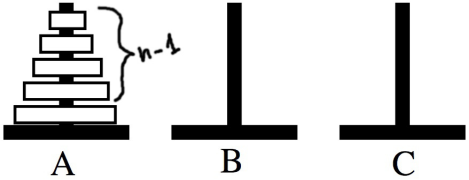


Tower of Hanoi rules

Recursive Solution

Algorithm: move n-disk tower from source to destination.

move $n-1$ disk tower from source to resting place
move 1 disk tower from source to destination
move $n-1$ disk from resting place to destination



Divide-and-conquer Approach

Divide and conquer is derived from the Latin saying *Divide et impera*.

In computer science, divide and conquer is an important algorithm design paradigm based on multi-branched recursion.

A divide and conquer algorithm works by recursively breaking down a problem into two or more sub-problems of the same (or related) type, until these become simple enough to be solved directly.

The solutions to the sub-problems are then combined to give a solution to the original problem.

Divide-and-conquer Approach

The following algorithms from the ones we covered so far employ this paradigm:

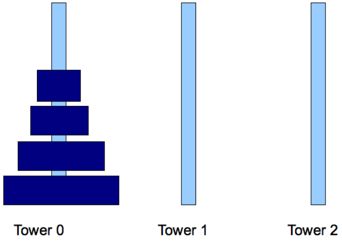
Binary Search (both versions) and Merge sort.

In-class Work

- Use Merge Sort to sort the following numbers: 5,1,6,2,8,3,9

  Show the graphical representation of the sort (use lecture slides)

- solve the Tower of Hanoi puzzle for four discs.


Tower 0
Tower 1
Tower 2