Outline

The Stack ADT

A Container Class for Last-In-First-Out Access

A stack is a **last in, first out** (**LIFO**) structure, i.e. a list-like container with access restricted to one end of the list (the top of the stack). One can

- push an item onto the stack
- pop an item off the stack (precondition: stack is not empty)
- Inspect the top position (precondition: stack is not empty)
- Obtain the current size of the stack.

The Stack ADT

Specification for a typical stack

```
class Stack:
   def __init__(self):
      """ post:  creates an empty LIFO stack"""
   def push(self,x):
      """post:  places x on top of the stack"""
   def pop(self):
      """pre:  self.size()>0
      post:  removes and returns the top element"""
   def top(self):
      """pre:  self.size()>0
      post:  returns the top element"""
   def size(self):
      """post:  returns the number of elements in the stack"""
```

Simple Stack Applications

Few Examples of Stack Applications

- graphical editors ("undo" operations)
- function calls ("nested" function calls)
- Evaluation of expressions
  example: $((x + y)/(2 * x) - 10 * z)$ - balance of grouping symbols

  See the code of Stack.py and next slide

Simple Stack Applications

Balanced Parentheses

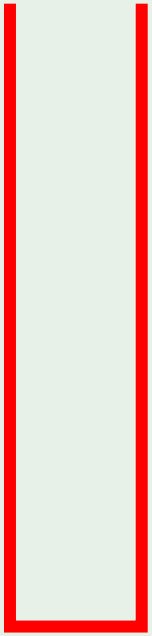
The following code checks if parentheses in the expression are balanced.

```
def parensBalance2(s):
   stack = Stack()
   for ch in s:
      if ch in "([{":  " push an opening marker "
         stack.push(ch)
      elif ch in ")]}":  " match closing "
         if stack.size() < 1:  " no pending open "
            return False
         else:
            opener = stack.pop()
            if opener+ch not in ["()", "[]", "{}"]:
               return False " not a matching pair"
   return stack.size() == 0 " everything matched?"
```

Simple Stack Applications

Balanced Parentheses

{[2 * (7 - 4) + 2] + 3} * 4



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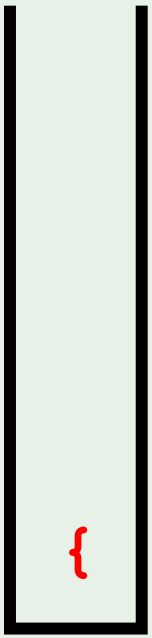

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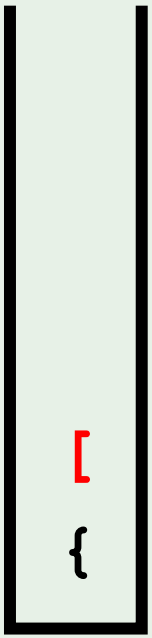

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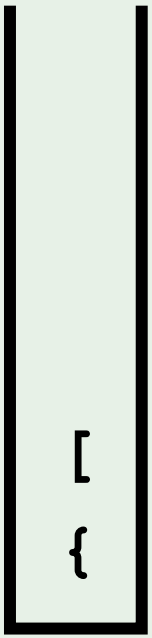

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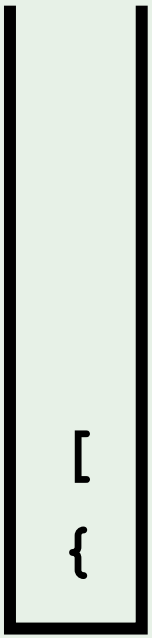

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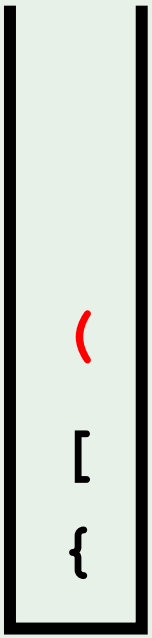

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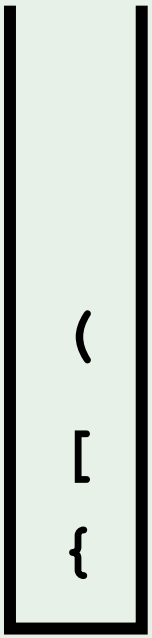

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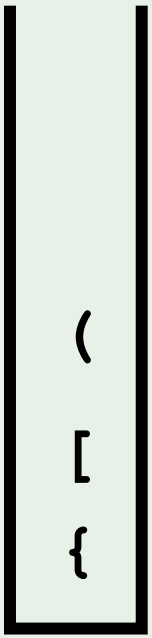

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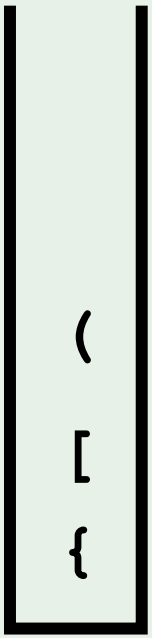

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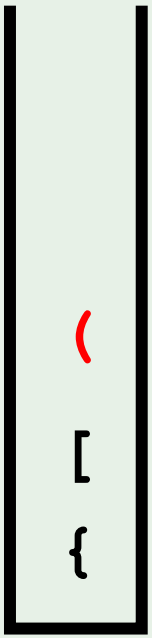

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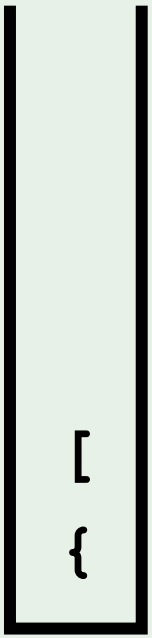

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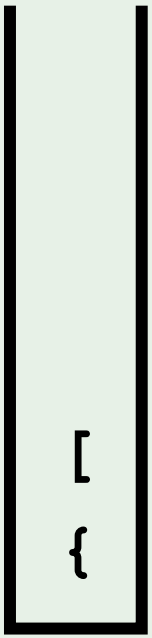

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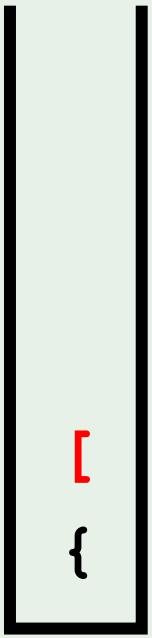

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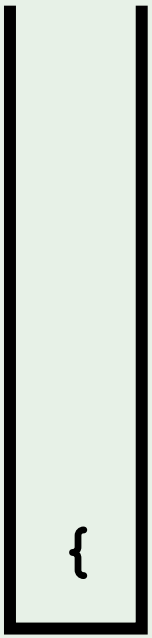

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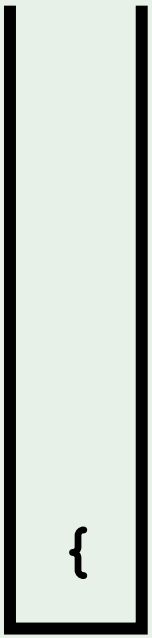

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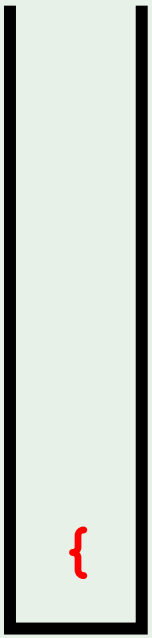

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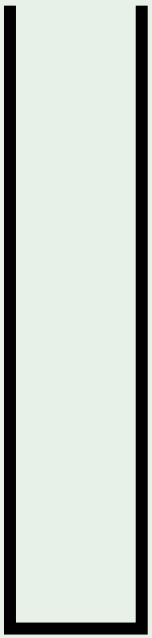

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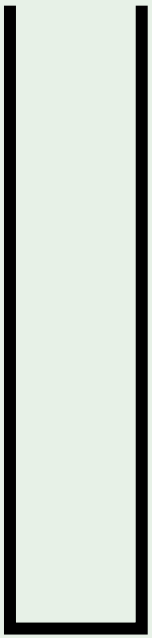


An Application: Expression Manipulation

Notation For Operations

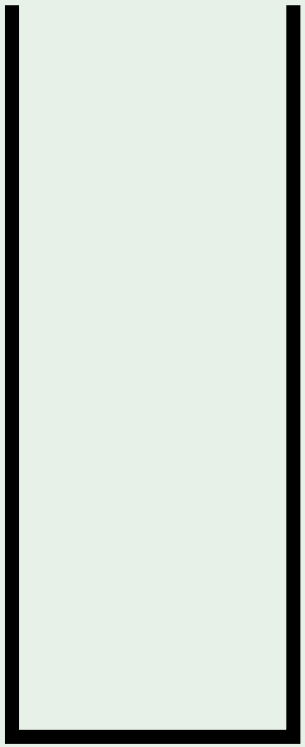
- infix notation: `(2 + 3) * 4`
  operators are between numbers
- prefix (Polish) notation: `* + 2 3 4`
- postfix (reverse Polish) notation: `2 3 + 4 *`
  advantage of two last ones: parentheses are not necessary to modify the order of operations.

  Postfix notation expressions can be evaluated easily using a stack:
  each time an operation is encountered, two numbers are popped off the stack, the operator is applied to those two numbers, and the result is pushed on the stack.

An Application: Expression Manipulation

Evaluating A Postfix Expression

3 4 5 + * 2 - 3 6 * +



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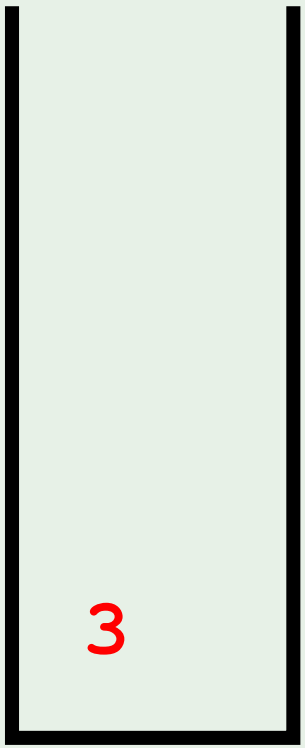

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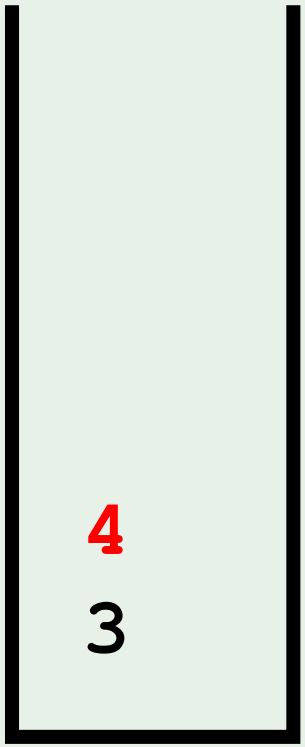

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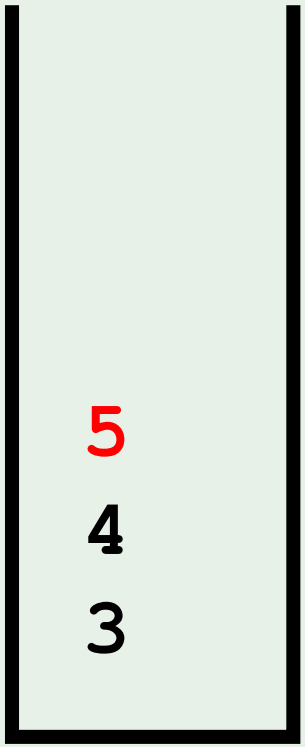

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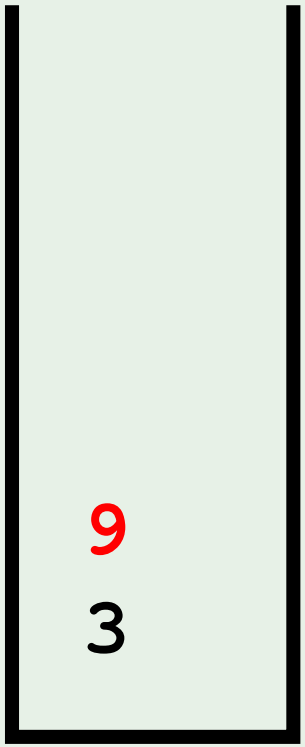

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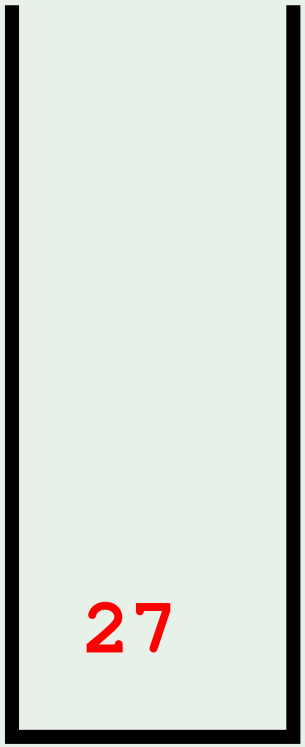

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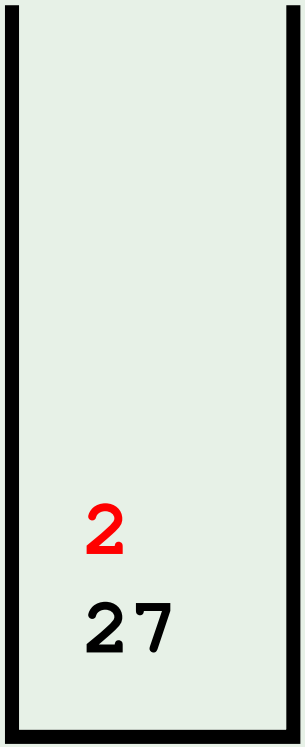

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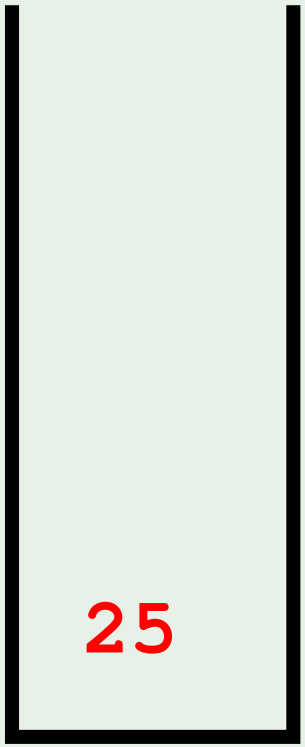

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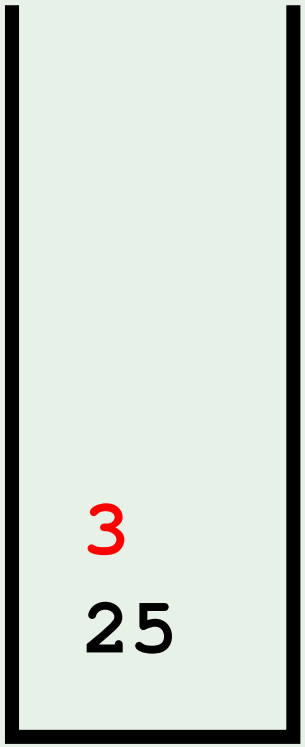

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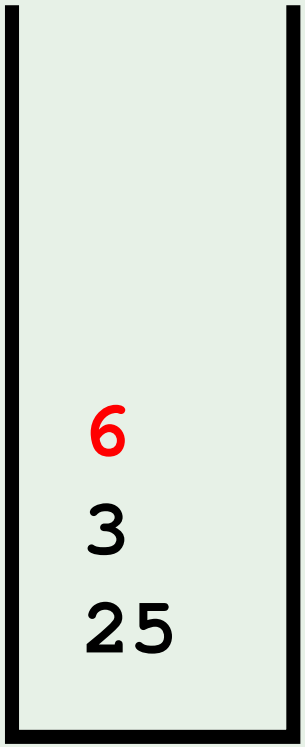

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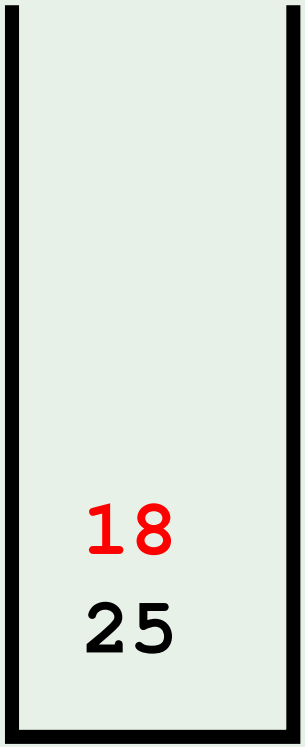

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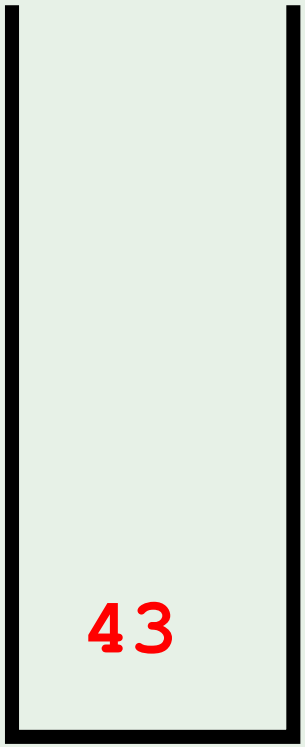

The Call Stack

Function Calls Can Be Nested

- function `A` calls function `B`
- function `B` returns
- function `A` continues

The Call Stack

Activation Records

- Function `A` is running, and calls function `B`.
- The local variables of function `A`, their current values, and where function `B` should return to are put into an activation record.
- The activation record is pushed onto the call stack which has been allocated for the program that is running.
- When function `B` returns, this record is popped off the call stack and used to continue running the program.

The Call Stack

Example

```
def A(x, y):
    1:   x2 = B(x)
    2:   y2 = B(y)
    3:   z = x2 + y2
    4:   return z
def B(n):  'squares n '
    5:   n2 = n * n
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def main():
    7:   a = 3
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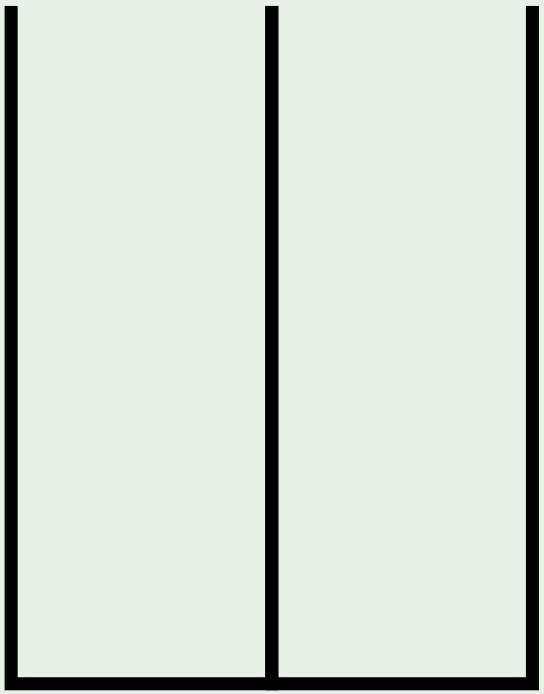

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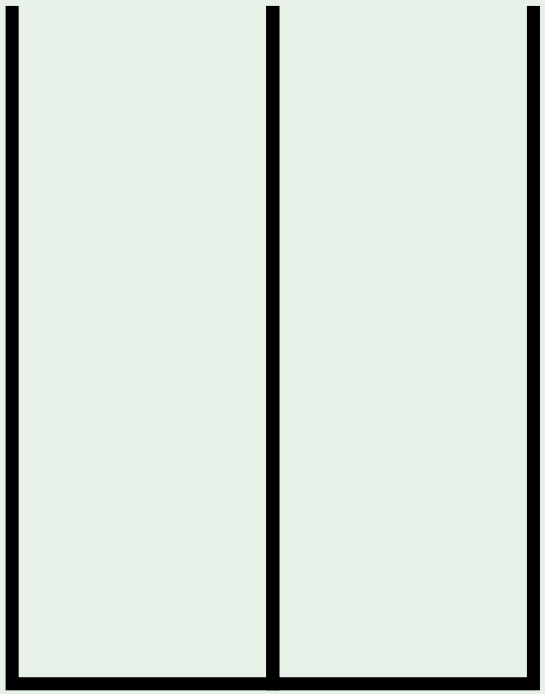

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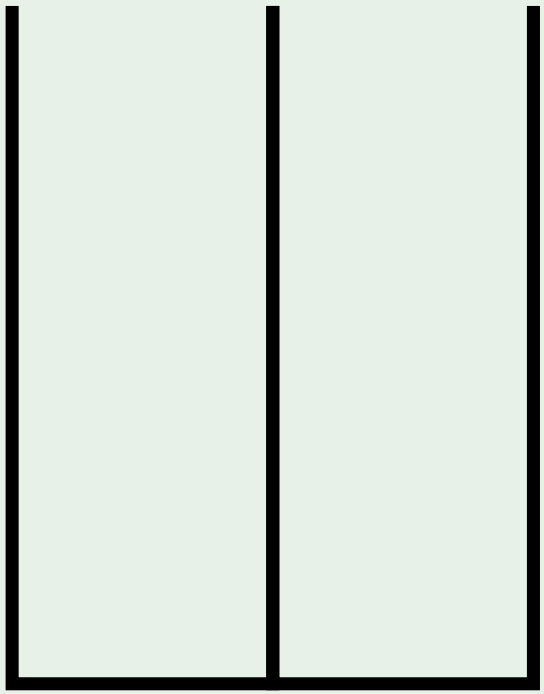

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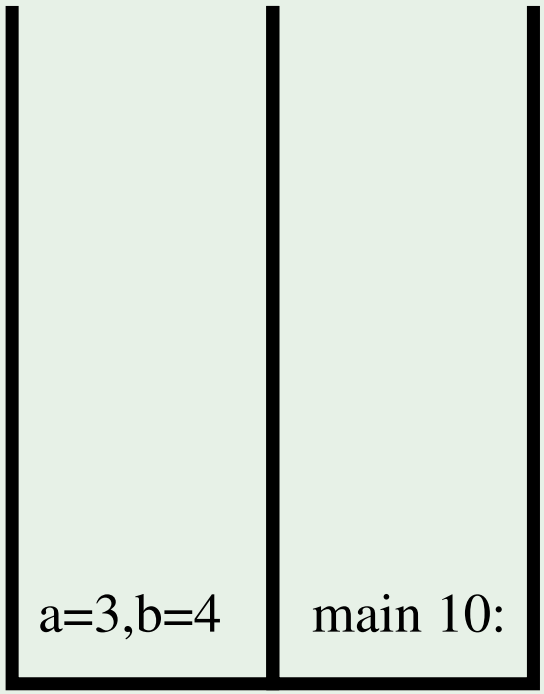

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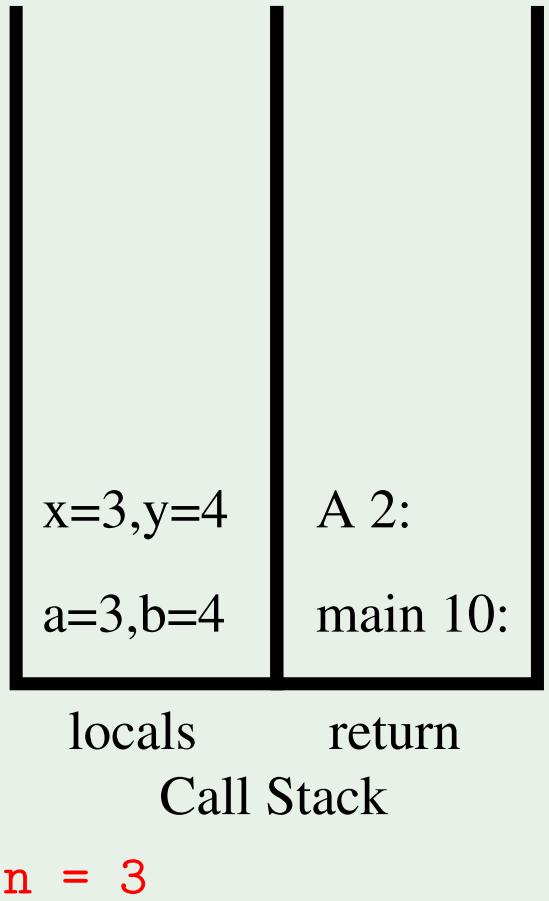

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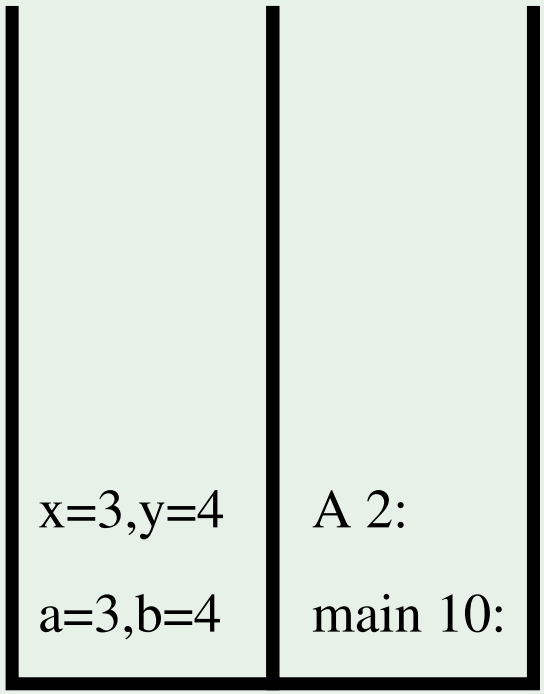

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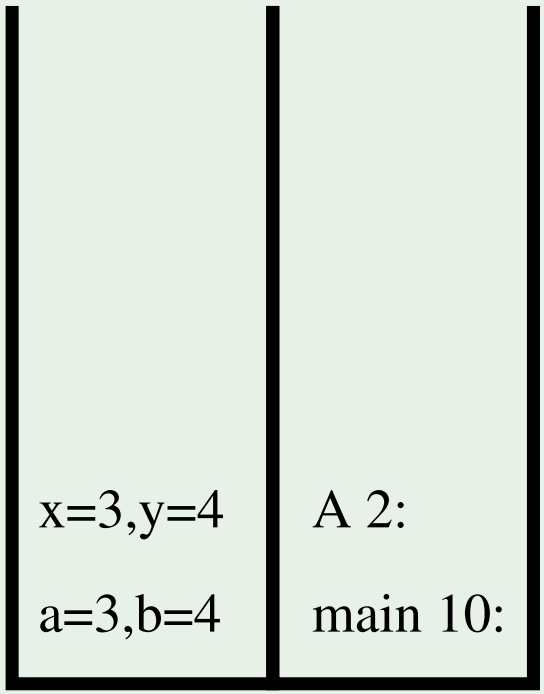

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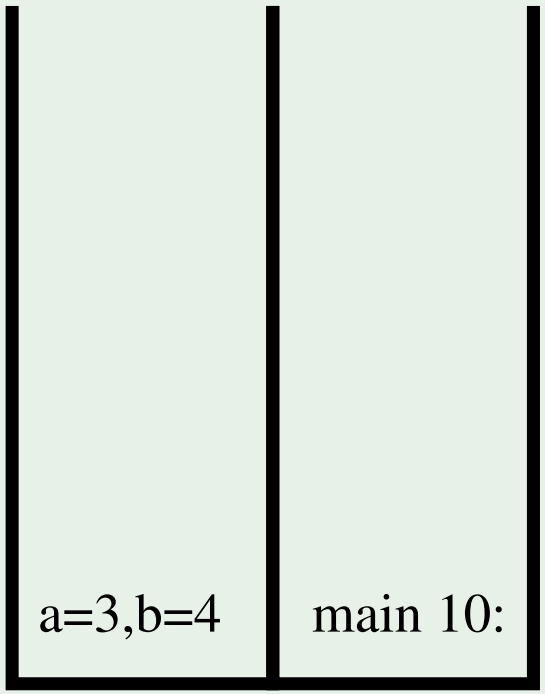

x = 3, y = 4, x2 = 9

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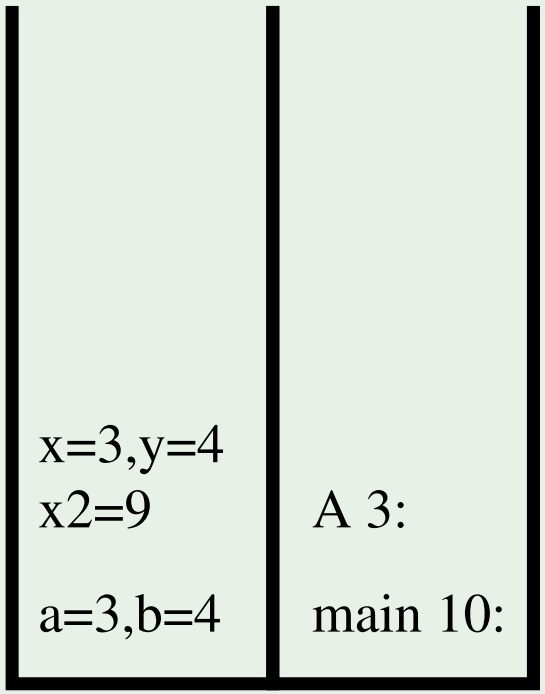
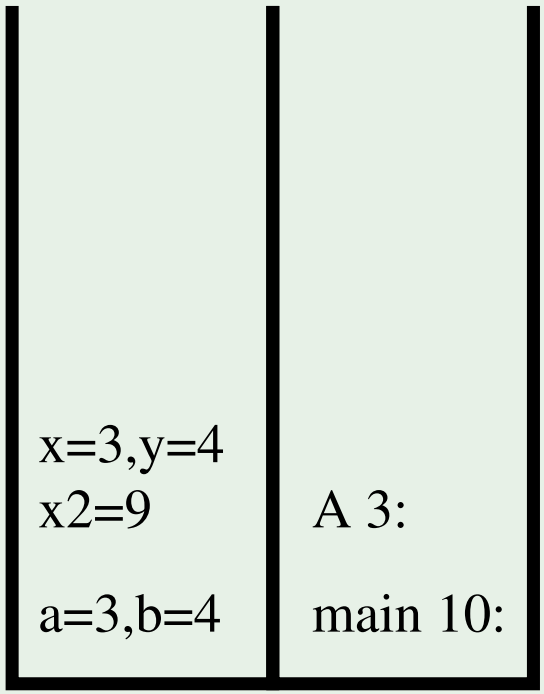


n = 4

The Call Stack

Example

```
def A(x, y):
   1:   x2 = B(x)
   2:   y2 = B(y)
   3:   z = x2 + y2
   4:   return z
def B(n):  'squares n '
   5:   n2 = n * n
   6:   return n2
def main():
   7:   a = 3
   8:   b = 4
   9:   c = A(a, b)
   10:   print(c)
   11:   return
```

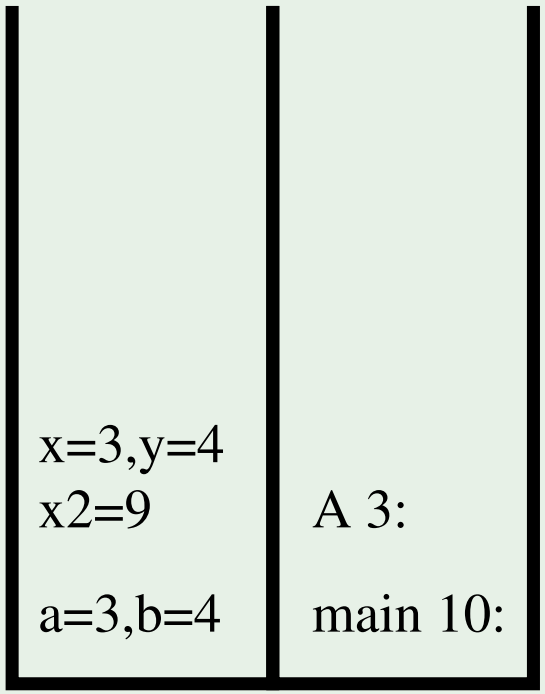


n = 4, n2 = 16

The Call Stack

Example

```
def A(x, y):
   1:   x2 = B(x)
   2:   y2 = B(y)
   3:   z = x2 + y2
   4:   return z
def B(n):  'squares n '
   5:   n2 = n * n
   6:   return n2
def main():
   7:   a = 3
   8:   b = 4
   9:   c = A(a, b)
   10:   print(c)
   11:   return
```

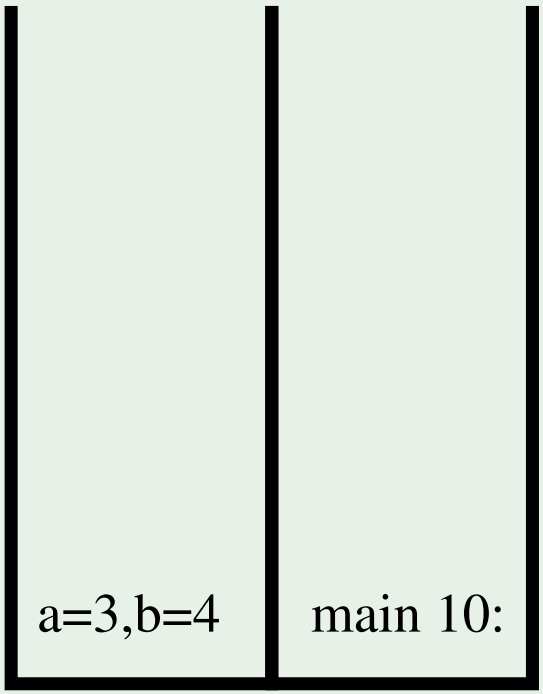


`n = 4, n2 = 16`

The Call Stack

Example

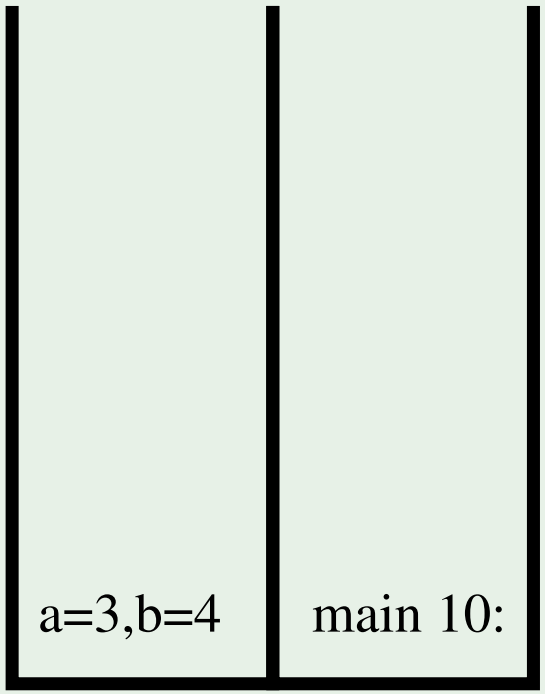
```
def A(x, y):
   1:   x2 = B(x)
   2:   y2 = B(y)
   3:   z = x2 + y2
   4:   return z
def B(n):  'squares n '
   5:   n2 = n * n
   6:   return n2
def main():
   7:   a = 3
   8:   b = 4
   9:   c = A(a, b)
   10:   print(c)
   11:   return
```



The Call Stack

Example

```
def A(x, y):
    1:   x2 = B(x)
    2:   y2 = B(y)
    3:   z = x2 + y2
    4:   return z
def B(n):  'squares n '
    5:   n2 = n * n
    6:   return n2
def main():
    7:   a = 3
    8:   b = 4
    9:   c = A(a, b)
    10:   print(c)
    11:   return
```

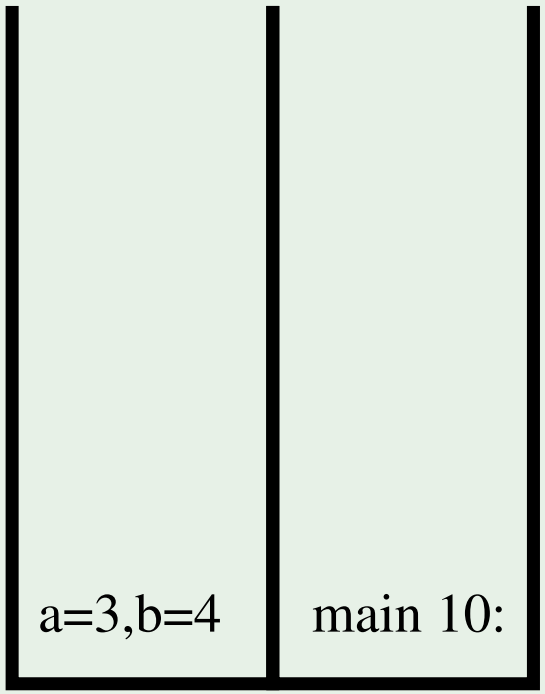


x=3, y=4, x2=9, y2=16, z=25

The Call Stack

Example

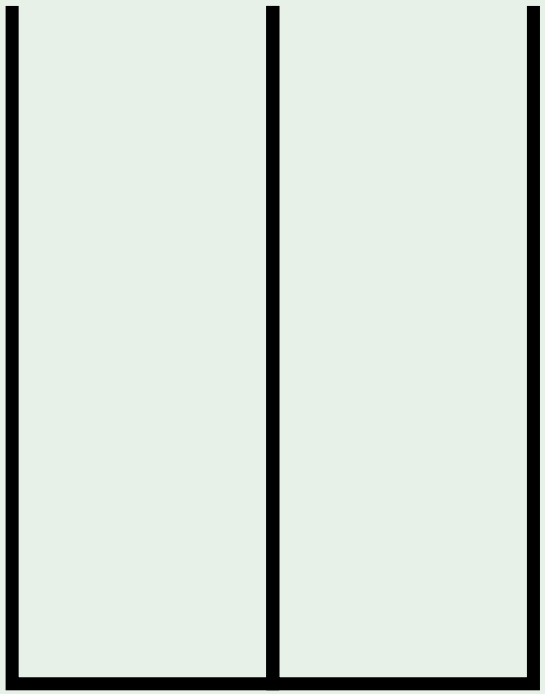
```
def A(x, y):
    1:   x2 = B(x)
    2:   y2 = B(y)
    3:   z = x2 + y2
    4:   return z
def B(n):  'squares n '
    5:   n2 = n * n
    6:   return n2
def main():
    7:   a = 3
    8:   b = 4
    9:   c = A(a, b)
    10:   print(c)
    11:   return
```



The Call Stack

Example

```
def A(x, y):
    1:   x2 = B(x)
    2:   y2 = B(y)
    3:   z = x2 + y2
    4:   return z
def B(n):  'squares n '
    5:   n2 = n * n
    6:   return n2
def main():
    7:   a = 3
    8:   b = 4
    9:   c = A(a, b)
    10:   print(c)
    11:   return
```



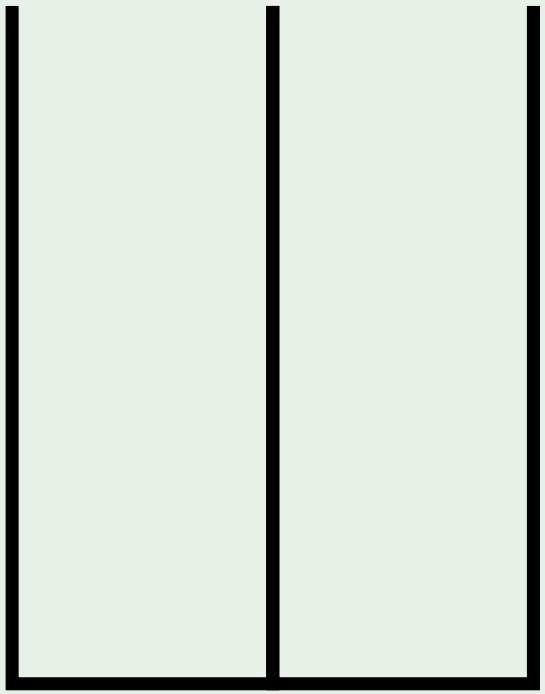
Call Stack

a = 3, b = 4, c = 25

The Call Stack

Example

```
def A(x, y):
    1:   x2 = B(x)
    2:   y2 = B(y)
    3:   z = x2 + y2
    4:   return z
def B(n):  'squares n '
    5:   n2 = n * n
    6:   return n2
def main():
    7:   a = 3
    8:   b = 4
    9:   c = A(a, b)
    10:   print(c)
    11:   return
```



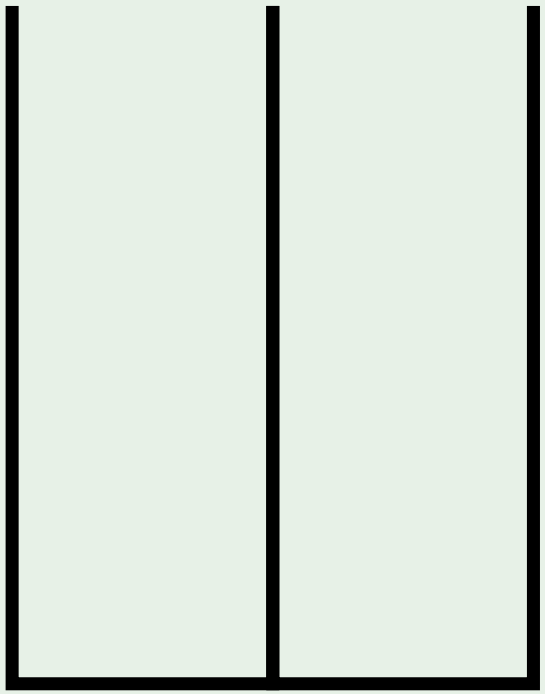
Call Stack

a = 3, b = 4, c = 25

The Call Stack

Example

```
def A(x, y):
    1:   x2 = B(x)
    2:   y2 = B(y)
    3:   z = x2 + y2
    4:   return z
def B(n):  'squares n '
    5:   n2 = n * n
    6:   return n2
def main():
    7:   a = 3
    8:   b = 4
    9:   c = A(a, b)
    10:   print(c)
    11:   return
```



Call Stack

a = 3, b = 4, c = 25

In-class work

- Re-write expression $7 * (2 + 5) - 3 * (6 - 7)$ in postfix notation
- re-write the expression 3 2 5 7 3 $-$ $+$ $*$ $-$ (it is in postfix notation) in infix notation (common way)
- Do *unit testing* of methods push and size in Stack.py.

  For example, to test the `push` function:
  `push` a value onto the stack, retrieve it immediately (using `pop` or `top`) and check whether the retrieved value is equal to the one you just pushed.