

1) Find the **running time $T(n)$** and the asymptotic running time (using Θ -notation and O -notation) of the following piece of code:

```
n=eval(input("Enter an integer number greater than 2:"))
for i in range(n):      n iterations
    print(i)           1 step
for j in range(n):      n iterations
    print(j)           1 step
```

$$T(n) = n+n = 2n \quad T(n) = \Theta(n)$$

2) Find the **running time $T(n)$** and the asymptotic running time (using Θ -notation and O -notation) of the following piece of code:

```
n=eval(input("Enter an integer number greater than 10:"))
for i in range(n):      n iterations
    for j in range(n):  n iterations
        print(i,"\\t",j) 1 step
```

$$T(n) = n*n*1 = n^2 \quad T(n) = \Theta(n^2)$$

3) Find the **running time $T(n)$** and the asymptotic running time (using Θ -notation and O -notation) of the following piece of code:

```
n=eval(input("Enter an integer number greater than 12:"))
while n>1:
    print(n)           1 step
    n = n//2          each time n becomes "twice less", until eventually it is <= 1, 2 steps
print(n)             1 step
```

$$T(n) = 3\log_2 n + 1 \quad T(n) = \Theta(\log n)$$

1) Copy the following program (you may omit the docstring):

```
def summation1(n):
    """ finds the sum (n+i)^2/i, where i runs from 1 to n

    pre: n in positive integer
    post: returns a positive integer number."""
    sum = 0
    for elem in list(range(n)):
        sum += (n+1+elem)**2/(elem+1)
    return sum
```

$$\sum_{i=1}^n \frac{(n+i)^2}{i}$$

2) run the defined procedure on different inputs, for example $n = 1, 2, 10$.
Write down the results.

```
4.0
17.0
547.8968253968254
```

3) Write, following the code of the program, each call of this procedure on inputs $n = 1, 2, 10$ as a sum of fractions, i.e. write which sum finds for procedure for each of these calls, but don't calculate it.

n=1 list: 0	n=2 list = 0,1	n=10 list = 0,1,2,3,4,5,6,7,8,9
sum: $0 + \frac{2^2}{1}$	sum: $0 + \frac{3^2}{1} + \frac{4^2}{2}$	sum: $0 + \frac{11^2}{1} + \frac{12^2}{2} + \frac{13^2}{3} + \frac{14^2}{4} + \frac{15^2}{5} + \frac{16^2}{6} + \frac{17^2}{7} + \frac{18^2}{8} + \frac{19^2}{9} + \frac{20^2}{10}$
as expected	as expected	as expected

4) find the running time of the procedure (depending on n), assuming that it takes one unit of time for each of math operations; the assignment operator and `range` function take also one time unit, and function `list` takes n time units.

```
sum = 0                                1 step/ time unit
for elem in list(range(n)):             range(n): 1 step;    list: n steps
n iterations
    sum += (n+1+elem)**2/(elem+1)
    ↑↑↑      ↑↑      ↑      7 steps → 7n steps
return sum                               1 step
                                         (we have n
                                         iterations,
                                         with 7 steps each)
```

Therefore, $T(n) = 1+1+n+7n+1 = 3+8n$

5) What is the order of growth (in terms of O and Θ)?

$O(n), \Theta(n)$

1) Copy the following program (you may omit the docstring):

```
def summation2(n):
    """ finds the sum 2^i/i, where i runs from 1 to n

    pre: n in positive integer
    post: returns a positive integer number. """
    sum = 0
    for elem in list(range(n)):
        sum += 2**(elem+1)/(elem+1)
    return sum
```

$$\sum_{i=1}^n \frac{2^i}{i}$$

2) run the defined procedure on different inputs, for example $n = 1, 2, 10$.
Write down the results.

```
2.0
4.0
237.30793650793652
```

3) Write, following the code of the program, each call of this procedure on inputs $n = 1, 2, 10$ as a sum of fractions, i.e. write which sum finds for procedure for each of these calls, but don't calculate it.

n=1 list: 0	n=2 list = 0,1	n=10 list = 0,1,2,3,4,5,6,7,8,9
sum: $0 + \frac{2^1}{1}$	sum: $0 + \frac{2^1}{1} + \frac{2^2}{2}$	sum: $0 + \frac{2^1}{1} + \frac{2^2}{2} + \frac{2^3}{3} + \frac{2^4}{4} + \frac{2^5}{5} + \frac{2^6}{6} + \frac{2^7}{7} + \frac{2^8}{8} + \frac{2^9}{9} + \frac{2^{10}}{10}$
<i>as expected</i>	<i>as expected</i>	<i>as expected</i>

4) find the running time of the procedure (depending on n), assuming that it takes one unit of time for each of math operations; the assignment operator and `range` function take also one time unit, and function `list` takes n time units.

```
sum = 0                                1 step/ time unit
for elem in list(range(n)): range(n): 1 step; list: n steps
n iterations
    sum += 2**(elem+1)/(elem+1)
    ↑↑↑↑↑ 6 steps → 6n steps
return sum 1 step                       (we have n
                                         iterations,
                                         with 6 steps each)
```

Therefore, $T(n) = 1+1+n+6n+1 = 3+7n$

5) What is the order of growth (in terms of O and Θ)?

$O(n), \Theta(n)$

1) Copy the following program (you may omit the docstring):

```
def summation3(n):
    """ finds the sum  $i^2/(i+1)$ , where  $i$  runs from 1 to  $n$ 

    pre:  $n$  in positive integer
    post: returns a positive integer number. """
    sum = 0
    for elem in list(range(n)):
        sum += (elem+1)**2/(elem+2)
    return sum
```

$$\sum_{i=1}^n \frac{i^2}{i+1}$$

2) run the defined procedure on different inputs, for example $n = 1, 2, 10$.
Write down the results.

```
0.5
1.8333333333333333
47.019877344877344
```

3) Write, following the code of the program, each call of this procedure on inputs $n = 1, 2, 10$ as a sum of fractions, i.e. write which sum finds for procedure for each of these calls, but don't calculate it.

n=1 list: 0	n=2 list = 0,1	n=10 list = 0,1,2,3,4,5,6,7,8,9
sum: $0 + \frac{1^2}{2}$	sum: $0 + \frac{1^2}{2} + \frac{2^2}{3}$	sum: $0 + \frac{1^2}{2} + \frac{2^2}{3} + \frac{3^2}{4} + \frac{4^2}{5} + \frac{5^2}{6} + \frac{6^2}{7} + \frac{7^2}{8} + \frac{8^2}{9} + \frac{9^2}{10} + \frac{10^2}{11}$
as expected	as expected	as expected

4) find the running time of the procedure (depending on n), assuming that it takes one unit of time for each of math operations; the assignment operator and `range` function take also one time unit, and function `list` takes n time units.

```
sum = 0
for elem in list(range(n)):
    sum += (elem+1)**2/(elem+2)
return sum
```

1 step/ time unit
range(n): 1 step; list: n steps
n iterations
↑↑↑↑↑ 6 steps → 6n steps (we have n iterations, with 6 steps each)
return sum 1 step

Therefore, $T(n) = 1+1+n+6n+1 = 3+7n$

5) What is the order of growth (in terms of O and Θ)?

$O(n)$, $\Theta(n)$