Chapter 2. Sections 2.1 and 2.2

Set is an unordered collection of objects.

$a \in A$ "a is an element of set A"
"a belongs to A"

$f \notin A$ "f is not an element of set A"
"f doesn't belong to A"

listing elements:
A = {1,2,3,4,5,6}
or
A = {1,2,3,...,20}

set builder notation:
A = { x | x is even positive integer}

Theorem 1
For every set S,
(1) $\varnothing \subseteq S$
(2) $S \subseteq S$

Theorem
$|\mathbf{A} \cup \mathbf{B}| = |\mathbf{A}| + |\mathbf{B}| - |\mathbf{A} \cap \mathbf{B}|$

Given a set S, the power set of S, P(S), is the set of all subsets of the set S.

Theorem
Given a set S, with n elements, the power set of S has 2^n elements, i.e.
$|P(s)|=2^n$

Some of the known sets:

N = { 0, 1, 2, 3, 4, ...} set of natural numbers
(in math: **W** – set of whole numbers)

Z = {...,-3, -2, -1, 0, 1, 2, 3, ...} set of integers

$\mathbf{Z}^+$ = { 1, 2, 3, 4, ...} set of positive integers

R – the set of real numbers (rational and irrational numbers)

Q = $\{ \frac{p}{q} \mid p \in Z, q \in Z, q \neq 0\}$ set of fractions (quotients)

Two sets are equal if and only if (iff) they have the same elements.

$$A = B \quad \text{iff} \quad \forall x (x \in A \leftrightarrow x \in B)$$

Empty set is a set that has no elements; denotation: $\varnothing$

Singleton set is a set that has exactly one element.

A is a subset of B, $A \subseteq B$, iff every element of A is also an element of B.

$$A \subseteq B \quad \text{iff} \quad \forall x(x \in A \rightarrow x \in B) \text{ is true}$$

A is a proper subset of B, $A \subset B$, iff A is a subset of B and set A and B are not equal.

$$A \subset B \quad \text{iff} \quad \forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A) \text{ is true}$$

The union of two sets A and B is the set that contains those elements that are either in A or in B, or in both.

$$\mathbf{A} \cup \mathbf{B} = \{ x \mid x \in \mathbf{A} \vee x \in \mathbf{B} \}$$

The intersection of two sets **A** and **B** is the set that contains those elements that are in both **A** and **B**.

$$\mathbf{A} \cap \mathbf{B} = \{ x \mid x \in \mathbf{A} \wedge x \in \mathbf{B} \}$$

The union of three sets **A**, **B**, and **C** is the set that contains those elements that are members of at least one of the sets.

$$\mathbf{A} \cup \mathbf{B} \cup \mathbf{C} = \{ x \mid x \in \mathbf{A} \vee x \in \mathbf{B} \vee x \in \mathbf{C} \}$$

The intersection of three sets **A**, **B**, and **C** is the set that contains those elements that are in each set of the collection.

$$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C} = \{ x \mid x \in \mathbf{A} \wedge x \in \mathbf{B} \wedge x \in \mathbf{C}\}$$

Two sets are disjoint if their intersection is $\varnothing$, i.e. $\mathbf{A} \cap \mathbf{B} = \varnothing$.

The difference of **A** and **B** is the set containing those elements that are in **A**, but not in **B**.

$$\mathbf{A} - \mathbf{B} = \{ x \mid x \in \mathbf{A} \wedge x \notin \mathbf{B} \}$$

The compliment of the set **A** is the set of all elements of the universe set *U* that are not the elements of set A.

$$\overline{\mathbf{A}} = \{ x \mid x \notin A \}$$

n-tuples are equal iff each corresponding pair of their elements is equal, i.e. $(a_1, a_2, ..., a_n)=(b_1, b_2, ..., b_n)$ iff $a_i=b_i$, for $i=1,2, ..., n$

Cartesian product of two sets, A and B, $A \times B$, is the set of all ordered pairs (a,b), where $a \in A$ and $b \in B$

$$A \times B = \{ (a,b) \mid a \in A \wedge b \in B\}$$

to check: $|A \times B|=|A| \cdot |B|$

Cartesian product of the sets $A_1, A_2, ..., A_n$, $A1 \times A2 \times A3 \times ... \times An$ $A1 \times A2 \times A3 \times ... \times An$ is the set of all ordered n-tuples $(a_1, a_2, ..., a_n)$, where $a_i \in A_i$ for $i = 1, 2, ..., n$

$$A_1 \times A_2 \times A_3 \times ... \times A_n = \{(a_1, a_2, ..., a_n) \mid a_i \in A_i, \text{ for } i = 1, 2, ..., n \}$$

Identity	***Name***
$A \cup \varnothing = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C)=(A \cap B) \cup (A \cap C)$ $A \cup (B \cap C)=(A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \varnothing$	Complement laws