BRONX COMMUNITY COLLEGE

of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

CSI 30

Chapter 1 review

1. Identify which of the statements is a proposition, and which is not.

   a) "I don't know what time is it now"

   b) "Don't touch her phone!"

2. Determine if the biconditional is true or false.

   $2^4 > 10$ if and only if the earth has shape of a plate.

3. Let p and q be the propositions

   p:"You drive over 65 miles per hour"

   q: "you get speeding ticket"

   Write these propositions using p and q and logical connectives.

   a) Driving over 65 miles is sufficient for getting a speeding ticket.

   b) You get a speeding ticket, but you didn't drive over 65 miles per hour

   c) You drive over 65 miles per hour, but you don't get a speeding ticket.

4. Let p and q be the propositions "The election is decided" and "The votes have been counted", respectively. Express each of these compound propositions as an English sentence.

   a) $\neg p \wedge q$ **b)** $p \leftrightarrow q$ **c)** $\neg p \rightarrow \neg q$

5. Use De Morgan's Laws to find the negation of each of the following statements.

   (a) Yoshiko knows Java and calculus

   (b) Rita will move to Oregon or Washington

6. Construct a truth table for the compound proposition

   $(\neg p \rightarrow \neg q) \wedge (q \leftrightarrow p)$

7. Show that the compound proposition

   $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

   is a tautology using truth table

8. Show that the compound proposition

   $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

   is a tautology without using truth tables, but using laws instead.

9. Show that

   $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$

   are logically equivalent

   (use truth tables first, then laws)

10. Determine the truth value of each of these statements if the domain of each variable consists of all integers. Explain your answer.

    a) $\exists n\ (n^2 = 81)$

    b) $\forall n\ (n^2 \neq n)$

11. Let $T(x)$ be the statement 'x has a Cable TV' and $C(x, y)$ be the statement 'x and y watch the same TV show'. The domain for the variables x and y consists of all students in your class. Use quantifiers and logical connectives to express each statement:
    a. Flora doesn't have a Cable TV

    b. Exactly one student in your class has a Cable TV.

    c. No one in the class has watched the same TV show with Joe.

12. Express the given statement using predicates, quantifiers, logical connectives, and mathematical operators. Domain: all real numbers.

    'There exists a real number such that if any real number is multiplied by it, we get 0 '

13. Let I(x) be the statement "x has an Internet connection" and C(x,y) be the statement "x and y have chatted over the Internet", where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.

    a) There are two students in your class who have not chatted with each other over the Internet.

    b) There is a student in your class who has chatted with everyone in your class over the Internet.

14. Rewrite the statement

    $\neg\exists y(\forall x\ \exists z\ T(x,y,z)\ \vee\ \exists x\forall z\ U(x,y,z))$

    so that negations appear only within predicates.

15. State which rule of inference is the basis of the following argument:

    "It is below freezing now. Therefore, it is either below freezing or raining now".

16. Indicate whether the argument is valid or invalid. For valid arguments, prove that the argument is valid using a truth table. For invalid arguments give truth values for the variables showing that the argument is not valid.

    $p \to q$

    $q \to r$

    $\neg r$

    ———

    $\neg p$

17. Indicate whether the argument is valid or invalid.

    "If n is a real number such that $n > 3$, then $n^2 > 9$. Suppose that $n <= 3$. Then $n^2 <= 9$ "

18. Use rules of inference to establish the conclusion.

    $(\neg p \vee \neg q) \to (r \wedge s)$

    $r \to t$

    $\neg t$

    ———

    p

Answers:

1. **a)** a proposition **b)** not a proposition (command)

2. **T** $\leftrightarrow$ **F** $\equiv$ **F**

3. **a)** $p \to q$ **b)** $q \wedge \neg p$ **c)** $p \wedge \neg q$

4. **a)** "The election is not decided, but the votes have been counted"
   b) "The election is decided if and only if the votes have been counted"
   c) "If the election is not decided then the votes haven't been counted"

5. **a)** "Yoshiko doesn't know Java or doesn't know calculus"
   b) "Rita won't move to Oregon and won't move to Washington"

6. The truth table for compound proposition $(\neg p \to \neg q) \wedge (q \leftrightarrow p)$:

p	q	$\neg p$	$\neg q$	$\neg p \to \neg q$	$q \leftrightarrow p$	$(\neg p \to \neg q) \wedge (q \leftrightarrow p)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	T	T	T

7. you can do it (you should see all Ts in the last column)!

8. $(\neg q \wedge (p \to q)) \to \neg p \equiv$
   $\equiv (\neg q \wedge (\neg p \vee q)) \to \neg p$ by law (11)
   $\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \to \neg p$ by Distributive law
   $\equiv ((\neg q \wedge \neg p) \vee F) \to \neg p$ by Negation law
   $\equiv (\neg q \wedge \neg p) \to \neg p$ by Identity law
   $\equiv \neg(\neg q \wedge \neg p) \vee \neg p$ by law (11)
   $\equiv (\neg\neg q \vee \neg\neg p) \vee \neg p$ by DeMorgan's law
   $\equiv q \vee p \vee \neg p$ by Double Negation law twice and omitting the parentheses
   $\equiv q \vee T$ by Negation law
   $\equiv T$ by Domination law

9. Let's start with a truth table first:

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$p \to (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Now let's use laws:

$(p \to q) \vee (p \to r) \equiv$
$\equiv (\neg p \vee q) \vee (\neg p \vee r)$ by law (11) twice
$\equiv \neg p \vee q \vee \neg p \vee r$ omitting parentheses
$\equiv \neg p \vee q \vee r$ by commutativity and law (5)
$\equiv p \to (q \vee r)$ by law (11) right to left

10. **a)** True, because we can provide such an n: if $n = 9$, then $n^2 = 81$

    b)False, when $n = 1$, $n^2 = n = 1$ - counterexample.

11. **a)** $\neg T(Flora)$ **b)** $\exists! x T(x)$

    c) $\neg \exists x C(x, Joe) \equiv \forall x \neg C(x, Joe)$

12. $\exists x \forall y (x \cdot y = 0)$

13. **a)** $\exists x \exists y \ \neg C(x, y)$

    b) $\exists x \forall y \ C(x, y)$

14. $\neg \exists y (\forall x \ \exists z \ T(x, y, z) \vee \exists x \forall z \ U(x, y, z)) \equiv \forall y (\exists x \ \forall z \ \neg T(x, y, z) \wedge \forall x \exists z \ \neg U(x, y, z))$

15. Let p: "it is below freezing now" and q: "it is raining now". Then we get:

    p

    —— *Addition inference rule*

    $p \vee q$

16. We need to derive $\neg p$. Let's enumerate premises (hypotheses):

    1.$p \to q$

    2.$q \to r$

    3.$\neg r$

    ————————

    4.$p \to r$ *by Hypothetical syllogism from* 1&2

5.$\neg p$ *by Modus tollens from* 4&3

17. Let domain be all real numbers, let $P(n)$: $n > 3$, and $Q(n)$: $n^2 > 9$. Therefore we have:

    1.$\forall x(P(x) \to Q(x)$

    2.$\neg P(x)$

    ——————————————— *valid or invalid?*

    3.$\neg Q(x)$

    It might look like *Modus Ponens* or *Modus Tollens* rule of inference, but it is none of them. The argument is invalid (***fallacy of denying the hypotheses***).

18. If this problem we will use rules of inference and laws. We need to conclude p. Let's enumerate the premises (hyphotheses). :

    1.$(\neg p \vee \neg q) \to (r \wedge s)$

    2.$r \to t$

    3.$\neg t$

    ———————————————

    4.$\neg(p \wedge q) \to (r \wedge s)$ by DeMorgan's law from 1. (right to left)

    5.$\neg r$ by Modus tollens from 2&3

    6.$\neg r \vee \neg s$ by Addition inference rule

    7.$\neg(r \wedge s)$ by DeMorgran's law from 6 (right to left)

    8.$\neg\neg(p \wedge q)$ by Modus tollens from 4&7

    9.$p \wedge q$ by Double negation law from 8

    10.p by Simplification inference rule from 9