BRONX COMMUNITY COLLEGE

of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

CSI 30

Test 2: Chapters 2 and 3

(Sample, Solutions)

Present all your work in order to get full credit.
Show solutions, do not give just answers.

1. For the sets $A = \{a, g, f, d, t\}$, $B = \{f, a, t, g\}$, $C = \{f, g, t\}$.
   Determine whether these statements are true or not:

   (a) $B \subseteq A$ **True**

   (b) $C \subseteq A$ **True**

   (c) $C \subseteq B$ **True**

2. Determine which statements are false or true

   (a) $\{0\} \subseteq \{0\}$ **True**

   (b) $\{0\} \in \{0\}$ **False**

   (c) $\{1, 2\} \subseteq \{1, \{1, 2\}, \{2\}, \{\{1\}, 2\}, 2\}$ **True**

   (d) $\{1\} \in \{1, \{1, 2\}, \{2\}, \{\{1\}, 2\}, 2\}$ **False**

3. Determine the cardinality of the set A=$\{x \in Z^+ | x$ is less than 13$\}$

   Solution: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, therefore $|A| = 12$

4. Find the powerset of $\{a, b, c\}$

   Answer: $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

   check: $|P(\{a, b, c\})| = 2^3 = 8$, the powerset given above has 8 elements.

5. For the sets $A = \{1, 2, 8\}$, $B = \{a, b, 8\}$, $C = \{o, m, 1\}$. Find

   (a) Cartesian Product $A \times B \times C$

   Answer: $A \times B \times C = \{(1, a, o), (1, a, m), (1, a, l), (1, b, o), (1, b, m), (1, b, l), (1, 8, o), (1, 8, m), (1, 8, l), (2$
   $(2, 8, l), (8, a, o), (8, a, m), (8, a, l), (8, b, o), (8, b, m), (8, b, l), (8, 8, o), (8, 8, m), (8, 8, l)\}$

   check: $|A \times B \times C| = |A| \cdot |B| \cdot |C| = 3 \cdot 3 \cdot 3 = 27$. There are exactly 27 elements in the given above set.

(b) Cartesian Product $B \times C$

Answer: $B \times C = \{(a, o), (a, m), (a, 1), (b, o), (b, m), (b, 1), (8, o), (8, m), (8, 1)\}$

(c) Cartesian Product $C \times B$

Answer: $C \times B = \{(o, a), (o, b), (o, 8), (m, a), (m, b), (m, 8), (1, a), (1, b), (1, 8)\}$

(d) $A \bigcap B$ **Answer**: $A \bigcap B = \{8\}$

(e) $B \bigcap C$ **Answer**: $B \bigcap C = \emptyset$

(f) $A \bigcup B \bigcup C$ **Answer**: $A \bigcup B \bigcup C = \{1, 2, 8, a, b, o, m\}$

(g) $A - C$ **Answer**: $A - C = \{2, 8\}$

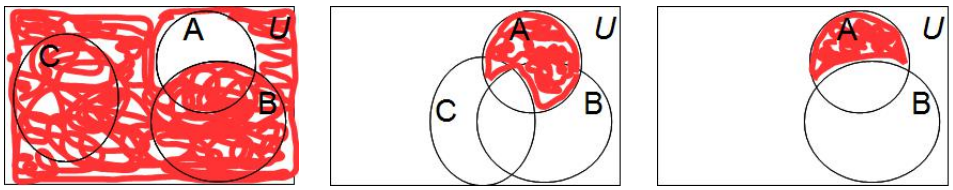
6. What can you say about sets A and B if we know that $A \bigcap B = A$?

   Answer: That $A \subseteq B$

7. Can we conclude that $A = B$ if we know that for sets A, B, C the following holds: $A \bigcup C = B \bigcup C$?

   Answer: No, we cannot. A and B might be different subsets of C. Then the equality above still holds, but sets are not equal.

8. For the given sets (see the Venn Diagramm) show on the Diagramm



a) $\overline{A - B}$ **b)** $A - (B \bigcap C)$ **c)** $\overline{B} \bigcap A$

9. For each function determine its range

   a) $g(x) = \lfloor x^2 \rfloor$, $g(x) : \mathtt{R} \to \mathtt{R}$

   Answer: range = {all non-negative integers} = $\{x | x \in \mathtt{Z} \text{ and } x \geq 0\}$

   b) $m(x) = \sqrt{2x}$, $m(x) : \mathtt{R}^+ \to \mathtt{R}^+$

   Answer: range = R^+

10. Determine whether the given functions are bijective. Are they invertible? If a function is invertible, what is its inverse?

    (a) $f : \{1, 2, 3, 4, 5\} \to \{a, b, c, d\}$ with $f(1) = a$, $f(2) = b$, $f(3) = a$, $f(4) = c$, $f(5) = d$

    Answer: not one-to-one, because $f(1) = f(3) = a$; onto, because every element from the codomain has preimage. Therefore, the function is not bijective, and not invertible.

(b) $f : R \to R$, $f(x) = -5x^2 + 8$

Answer: not one-to-one, because $f(-1) = f(1) = 3$; not onto, because $f(x) \leq 8$, therefore for $f(x) = 9$, there is no preimage x. The given function is not bijective, and not invertible.

11. Let $f(x) = 2x + 3$, $g(x) = x - 5$, and $h(x) = x^2$. $f : \mathtt{R} \to \mathtt{R}$, $g : \mathtt{R} \to \mathtt{R}$, $h : \mathtt{R} \to \mathtt{R}$

Find

a) $(fg)(x)$ **Answer**: $(fg)(x) = 2x^2 - 7x - 15$

b) $(g + h)(x)$ **Answer**: $(g + h)(x) = x^2 + x - 5$

c) $(f \circ h)(x)$ **Answer**: $(f \circ g)(x) = 2x^2 + 3$