BRONX COMMUNITY COLLEGE

of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

CSI 30 Sample Test 1 solutions

Present all your work in order to get full credit.
Show solutions when asked, do not give answers only.

1. Determine whether each of the following sentences is a proposition or not. If the sentence is a proposition, then write its negation.

   a) It is raining outside.

   Answer: it is a proposition. Its negation: "It isn't raining outside"

   b) What time is it now?

   Answer: it is a question, not a proposition (we cannot state whether it is True or False).

2. Construct a truth table for the compound proposition $(\neg p \vee q) \wedge (\neg r \to q)$

   Answer:

p	q	r	$\neg p$	$\neg r$	$\neg p \vee q$	$\neg r \to q$	$(\neg p \vee q) \wedge (\neg r \to q)$
T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	F	F	T	F
T	F	F	F	T	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	T	T	F	F

3. Show that the conditional statement $\neg p \to (p \to q)$ is a tautology by using truth tables

   Answer:

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
1	1	0	1	1
1	0	0	0	1
0	1	1	1	1
0	0	1	1	1

   The last column has 1's only (stands for True), therefore, no matter what are the values of p and q, the compound proposition $\neg p \to (p \to q)$ is always True, therefore it is a tautology.

4. Show that the compound proposition $\neg(p \to q) \to \neg q$ is a tautology without using truth tables.

   Solution:

   $\neg(p \to q) \to \neg q \equiv$

   $\equiv \neg\neg(p \to q) \vee \neg q \equiv$ *by law (11)*

   $\equiv (p \to q) \vee \neg q \equiv$ *by Double negation law (2)*

   $\equiv \neg p \vee q \vee \neg q \equiv$ *by law (11)*

$\equiv \neg p \vee T \equiv$ *by Negation law (8)*

$\equiv T$ *by Domination law (6)*

5. Show that the following compound propositions are equivalent using laws.

   $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

   Solution: $p \leftrightarrow q \equiv$

   $\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ by law (12)

   $\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ by law (11)

   $\equiv [(\neg p \vee q) \wedge (\neg q)] \vee [(\neg p \vee q) \wedge p]$ by Distributive law (10)

   $\equiv [(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \vee [(\neg p \wedge p) \vee (q \wedge p)]$ by Distributive law (10) twice

   $\equiv [(\neg p \wedge \neg q) \vee F] \vee [F \vee (q \wedge p)]$ by Negation law (8) twice

   $\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$ by Identity law (7) twice

   $\equiv (q \wedge p) \vee (\neg p \wedge \neg q)$ by Commutative law (7)

6. Let p, q, and r be the propositions:

   p: "Grizzly bears have been seen in the area."

   q: "Hiking is safe on the trail."

   r: "Berries are ripe along the trail."

   Write compound propositions using p,q, and r and logical connectives $\vee$, $\wedge$, $\neg$, $\rightarrow$, and $\leftrightarrow$.

   a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

   Answer: $r \wedge \neg p$

   b) Hiking is safe on the trail if and only if berries are not ripe along the trail or the grizzly bears have not been seen in the area.

   Answer: $q \leftrightarrow (\neg r \vee \neg p)$

   c) If berries are ripe along the trail then hiking along the trail is not safe.

   Answer: $r \rightarrow \neg q$

7. Determine the truth value of each of these statements if the domain of each variable consists of all integers. Provide explanation for your answer.

   a) $\forall x \exists y (xy \geq 0)$

   Answer: True, because for any given value of x we'll be able to find at least one value of y, such that their product will be positive.

   If we are given a negative value for x, then taking any negative value for y will make their product positive. If we are given a negative value for x, then taking any positive value for x will make their product positive. And if x is zero, then no matter what is the value of y, their product will be zero.

   b) $\exists ! x (5x - 1 = 2x + 4)$

   Answer: False.

   Let's solve the equation: $5x - 1 = 2x + 4$, or $3x = 5$ or $x = \dfrac{5}{3}$ - this is the only (unique) value of x, such that the equation is True, but our domain is *all integers*. Therefore, there is no solution to this equation (for the set of integers).

8. Use quantifiers and logical connectives to translate English statements into logical expressions. The domain of discourse is all people.

   a) No one is perfect.

   b) Not everyone is perfect.

   c) At least one of your friends is perfect.

   Solution:

   Let $P(x)$: "x is perfect" and $Q(x)$: "x is my friend", then

   a) $\neg\exists x P(x) \equiv \forall x \neg P(x)$

   b) $\exists x \neg P(x)$

   c) $\exists x(Q(x) \land P(x))$

9. Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

   $\forall x \exists y \forall z((\neg P(x, y) \land Q(x, z)) \to \neg T(y))$

   Solution:

   $\neg[\forall x \exists y \forall z((\neg P(x, y) \land Q(x, z)) \to \neg T(y))] =$

   $\exists x \forall y \exists z \neg((\neg P(x, y) \land Q(x, z)) \to \neg T(y)) =$

   $\exists x \forall y \exists z \neg(\neg(\neg P(x, y) \land Q(x, z)) \lor \neg T(y)) =$

   $\exists x \forall y \exists z \neg(\neg\neg P(x, y) \lor \neg Q(x, z) \lor \neg T(y)) =$

   $\exists x \forall y \exists z \neg(P(x, y) \lor \neg Q(x, z) \lor \neg T(y)) =$

   $\exists x \forall y \exists z(\neg P(x, y) \land \neg\neg Q(x, z) \land \neg\neg T(y)) =$

   $\exists x \forall y \exists z(\neg P(x, y) \land Q(x, z) \land T(y))$

10. Prove that the argument is valid by replacing each proposition with a variable to obtain the form of the argument. Then use the rules of inference and laws, if necessary, to prove that the form is valid.

    " If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material"''

    Solution: Let

    p: "I work on night on this homework"

    q: "I can answer all the exercises"

    r: "I will understand the material"

    Then we get:

    1.$p \to q$

    2.$q \to r$

    ————————— by Hypothetical syllogism

    3.$p \to r$