Logic Puzzles

Puzzles that can be solved using logical reasoning are called logic puzzles.

Raymond Smullyan posed lots of logic puzzles (read page 14).

Puzzle 1:
An island has two kinds of inhabitants:
knights – who always tell true, and
knaves – their opposites, who always lie
We encounter two people (A and B). Determine, if possible, what A and B are, if

a) A says: “at least one of us is a knave”
B says nothing.

b) A says: “we are both knaves”
B says nothing

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A says: $\neg p \lor \neg q$.

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Let p: "A is a knight"
q: "B is a knight"

A says: $\neg p \vee \neg q$.
<u>Let's assume</u> that A is a knight, then we have $p \wedge (\neg p \vee \neg q) \equiv$
we know that p is True (from our assumption): $\equiv T \wedge (F \vee \neg q)$

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For the entire compound proposition to be True, $\neg q$ must be True!

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Therefore, when p is True and q is False, the $p \wedge (\neg p \vee \neg q)$ is True.

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A is a knight, and B is a knave.

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Since p is False (from our assumption), we get $\equiv T \wedge \neg (T \wedge \neg q)$

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Therefore, when p is False and q is True, the $\neg p \wedge \neg(\neg p \wedge \neg q)$ is True.
A is a knave, and B is a knight.