CSI 30 Fall 2021
HW 5 answers and solutions

Section 1.9

Exercise 1.9.1: Which logical expressions with nested quantifiers are propositions?

The table below shows the value of a predicate M(x, y) for every possible combination of values of the variables x and y. The domain for x and y is {1, 2, 3}. The row number indicates the value for x and the column number indicates the value for y.
For example M(1, 2) = F because the value in row 1, column 2, is F.

M	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Indicate whether each of the logical expressions is a proposition. If so, indicate whether the proposition is true or false.
(a) M(1, 1)

Proposition. True.

(c) ∃x M(x, 3)

Proposition. True: for x = 1 or x = 2, M(x, 3) is true.

(e) ∃x ∀y M(x, y)

Proposition. False: there is no x such that M(x, 1), M(x, 2) and M(x, 3) are all true.

Exercise 1.9.2: Truth values for statements with nested quantifiers - small finite domain.

The tables below show the values of predicates P(x, y), Q(x, y), and S(x, y) for every possible combination of values of the variables x and y. The row number indicates the value for x and the column number indicates the value for y. The domain for x and y is {1, 2, 3}.

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

Indicate whether each of the quantified statements is true or false.

(g) ∀x ∀y P(x, y)

False. For example, when x = y = 3, P(x, y) is false.

(h) $\exists x \exists y Q(x, y)$

True. For example, when $x = 3$ and $y = 1$, $Q(x, y)$ is true.
(i) $\forall x \forall y \neg S(x, y)$

True. For every value for x and y, $\neg S(x, y)$ is true.

Exercise 1.9.3: Truth values for mathematical expressions with nested quantifiers.

Determine the truth value of each expression below. The domain is the set of all real numbers.
(b) $\exists x \forall y (xy = 0)$

True. If $x = 0$, then for all y, $xy = 0$.

(f) $\exists x \exists y \exists z (x^2 + y^2 = z^2)$

True. One example is $x = 3$, $y = 4$, $z = 5$.

(h) $\forall x \exists y (x < 0 \vee y^2 = x)$

True.

If $x < 0$, then the statement is True;

if $x \geq 0$, then there is a y such that $y^2 = x$: we can extract the square root $y = \sqrt{x}$

Exercise 1.9.4: De Morgan's law and nested quantifiers.

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(b) $\forall x \exists y (P(x, y) \wedge Q(x, y))$

$\exists x \forall y (\neg P(x, y) \vee \neg Q(x, y))$

(d) $\exists x \forall y (P(x, y) \leftrightarrow P(y, x))$

$\forall x \exists y ((P(x, y) \wedge \neg P(y, x)) \vee (\neg P(x, y) \wedge P(y, x)))$