CSI30 Fall 2021 HW 3 Solutions to HW problems

Section 1.4

1.4.1: Proving tautologies and contradictions.
Show whether each logical expression is a tautology, contradiction or neither.

(d)
$(p \rightarrow q) \lor p$

Tautology. Every truth value in the column for $(p \rightarrow q) \lor p$ is T.

p	q	$(p \rightarrow q) \lor p$
T	T	T
T	F	T
F	T	T
F	F	T

(e)

$(\neg p \lor q) \leftrightarrow (p \land \neg q)$

Contradiction. Every truth value in the column for $(\neg p \lor q) \leftrightarrow (p \land \neg q)$ is F.

p	q	$(\neg p \lor q) \leftrightarrow (p \land \neg q)$
T	T	F
T	F	F
F	T	F
F	F	F

(f)

$(\neg p \lor q) \leftrightarrow (\neg p \land q)$

Neither. If $p = F$ and $q = T$, then $(\neg p \lor q) \leftrightarrow (\neg p \land q)$ is true. If $p = T$ and $q = T$, then $(\neg p \lor q) \leftrightarrow (\neg p \land q)$ is false.

1.4.2 Truth tables to prove logical equivalence.
Use truth tables to show that the following pairs of expressions are logically equivalent.

(c)
$\neg p \to q$ and $p \lor q$

p	q	$\neg p \to q$	$p \lor q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

The columns for $\neg p \to q$ and $p \lor q$ are the same.

1.4.3: Proving two logical expressions are not logically equivalent.
Prove that the following pairs of expressions are not logically equivalent.

(c)
$(p \to q) \land (r \to q)$ and $(p \land r) \to q$

When p = T, r = F, and q = F (or when p = F, r = T, and q = F), then $(p \land r) \to q$ is true and $(p \to q) \land (r \to q)$ is false.

(d)

$p \land (p \to q)$ and $p \lor q$

When p = T and q = F (or when p = F and q = T), then $p \land (p \to q)$ is false and $p \lor q$ is true.

1.4.4: Proving whether two logical expressions are equivalent.
Determine whether the following pairs of expressions are logically equivalent. Prove your answer. If the pair is logically equivalent, then use a truth table to prove your answer.

(d)
$p \land (p \to q)$ and $p \land q$

Logically equivalent. The columns for $p \land (p \to q)$ and $p \land q$ are the same.

p	q	$p \land (p \to q)$	$p \land q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Section 1.5

1.5.1: Label the steps in a proof of logical equivalence.
Below are several proofs showing that two logical expressions are logically equivalent. Label the steps in each proof with the law used to obtain each proposition from the previous proposition. The first line in the proof does not have a label.

(c)

$r \lor (\neg r \to p)$
$r \lor (\neg\neg r \lor p)$
$r \lor (r \lor p)$
$(r \lor r) \lor p$
$r \lor p$

$r \lor (\neg r \to p)$	
$r \lor (\neg\neg r \lor p)$	Conditional identity
$r \lor (r \lor p)$	Double negation law
$(r \lor r) \lor p$	Associative law
$r \lor p$	Idempotent law

1.5.2: Using the laws of logic to prove logical equivalence.
Use the laws of propositional logic to prove the following:

(e)

$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

$(p \to r) \lor (q \to r)$	
$(\neg p \lor r) \lor (q \to r)$	Conditional identity
$(\neg p \lor r) \lor (\neg q \lor r)$	Conditional identity
$\neg p \lor (r \lor (\neg q \lor r))$	Associative law
$\neg p \lor ((r \lor \neg q) \lor r)$	Associative law
$\neg p \lor ((\neg q \lor r) \lor r)$	Commutative law
$\neg p \lor (\neg q \lor (r \lor r))$	Associative law
$\neg p \lor (\neg q \lor r)$	Idempotent law
$(\neg p \lor \neg q) \lor r$	Associative law
$\neg(p \land q) \lor r$	De Morgan's law
$(p \land q) \to r$	Conditional identity

(f)

$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	
$(\neg p \land \neg(\neg p \land q))$	De Morgan's law
$(\neg p \land (\neg\neg p \lor \neg q))$	De Morgan's law
$(\neg p \land (p \lor \neg q))$	Double negation law
$(\neg p \land p) \lor (\neg p \land \neg q)$	Distributive law
$F \lor (\neg p \land \neg q)$	Complement law
$\neg p \land \neg q$	Identity law

(g)

$(p \land q \land \neg r) \lor (p \land \neg q \land \neg r) \equiv p \land \neg r$

$(p \land q \land \neg r) \lor (p \land \neg q \land \neg r)$	
$(p \land \neg r \land q) \lor (p \land \neg q \land \neg r)$	Commutative law
$(p \land \neg r \land q) \lor (p \land \neg r \land \neg q)$	Commutative law
$((p \land \neg r) \land q) \lor ((p \land \neg r) \land \neg q)$	Associative law
$(p \land \neg r) \land (q \lor \neg q)$	Distributive law
$(p \land \neg r) \land T$	Complement law
$p \land \neg r$	Identity law

(h)

$p \leftrightarrow (p \land r) \equiv \neg p \lor r$

$p \leftrightarrow (p \land r)$	
$(p \to (p \land r)) \land ((p \land r) \to p)$	Conditional identity
$(p \to (p \land r)) \land (\neg(p \land r) \lor p)$	Conditional identity
$(p \to (p \land r)) \land ((\neg p \lor \neg r) \lor p)$	De Morgan's law
$(p \to (p \land r)) \land ((\neg r \lor \neg p) \lor p)$	Commutative law
$(p \to (p \land r)) \land (\neg r \lor (\neg p \lor p))$	Associative law
$(p \to (p \land r)) \land (\neg r \lor T)$	Complement law
$(p \to (p \land r)) \land T$	Domination law
$p \to (p \land r)$	Identity law
$\neg p \lor (p \land r)$	Conditional identity
$(\neg p \lor p) \land (\neg p \lor r)$	Distributive law
$T \land (\neg p \lor r)$	Complement law
$\neg p \lor r$	Identity law

1.5.3: Using the laws of logic to prove tautologies.
Use the laws of propositional logic to prove that each statement is a tautology.

(c)
$\neg r \lor (\neg r \to p)$

Prove that $\neg r \lor (\neg r \to p) \equiv T$.

$\neg r \lor (\neg r \to p)$	
$\neg r \lor (\neg\neg r \lor p)$	Conditional identity
$\neg r \lor (r \lor p)$	Double negation law
$(\neg r \lor r) \lor p$	Associative law
$T \lor p$	Complement law
T	Domination law