BRONX COMMUNITY COLLEGE
of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

CSI 30 Discrete Mathematics I

Final Exam Review Questions

Logic and Proofs

1. Identify which of the statements is a proposition, and which is not.

   a) "I don't know what time is it now"

   b) "Don't touch her phone!"

2. Determine if the biconditional is true or false.

   $2^4 > 10$ if and only if the earth has shape of a plate.

3. Let p and q be the propositions

   p:"You drive over 65 miles per hour"

   q: "you get speeding ticket"

   Write these propositions using p and q and logical connectives.

   a) Driving over 65 miles is sufficient for getting a speeding ticket.

   b) You get a speeding ticket, but you didn't drive over 65 miles per hour

   c) You drive over 65 miles per hour, but you don't get a speeding ticket.

4. Let p and q be the propositions "The election is decided" and "The votes have been counted", respectively. Express each of these compound propositions as an English sentence.

   a) $\neg p \wedge q$ **b)** $p \leftrightarrow q$ **c)** $\neg p \rightarrow \neg q$

5. Use De Morgan's Laws to find the negation of each of the following statements.

   (a) Yoshiko knows Java and calculus

   (b) Rita will move to Oregon or Washington

6. Construct a truth table for the compound proposition

   $(\neg p \rightarrow \neg q) \wedge (q \leftrightarrow p)$

7. Show that the compound proposition

   $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

   is a tautology using truth table

8. Show that the compound proposition

   $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

   is a tautology without using truth tables, but using laws instead.

9. Show that

   $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

   are logically equivalent

   (use truth tables first, then laws)

10. Determine the truth value of each of these statements if the domain of each variable consists of all integers. Explain your answer.

    a) $\exists n \ (n^2 = 81)$

    b) $\forall n \ (n^2 \neq n)$

11. Let $T(x)$ be the statement 'x has a Cable TV' and $C(x, y)$ be the statement 'x and y watch the same TV show'. The domain for the variables x and y consists of all students in your class. Use quantifiers and logical connectives to express each statement:
    a. Flora doesn't have a Cable TV

    b. Exactly one student in your class has a Cable TV.

    c. No one in the class has watched the same TV show with Joe.

12. Express the given statement using predicates, quantifiers, logical connectives, and mathematical operators. Domain: all real numbers.

    'There exists a real number such that if any real number is multiplied by it, we get 0 '

13. Let I(x) be the statement "x has an Internet connection" and C(x,y) be the statement "x and y have chatted over the Internet", where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.

    a) There are two students in your class who have not chatted with each other over the Internet.

    b) There is a student in your class who has chatted with everyone in your class over the Internet.

14. Rewrite the statement

    $\neg\exists y(\forall x \ \exists z \ T(x, y, z) \ \vee \ \exists x \forall z \ U(x, y, z))$

    so that negations appear only within predicates.

Sets

1. List the members of the following set.

   $\{x | x$ `is an integer such that` $x^2 < 25\}$

2. For sets $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 3, 5\}$, and $C = \{1, 3, 5, 7, 9, 11\}$, determine which of the following statements are true or false.

   a) $A \subseteq B$

   b) $A \subseteq C$

   c) $B \subseteq C$

3. Determine whether the following statements are true or false.

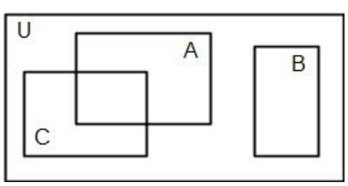
   a) $3 \in \{\emptyset, 3, \{3\}\}$

   b) $\{3, 1\} \subseteq \{1, \{2\}, \{3\}, \{1, 3\}\}$

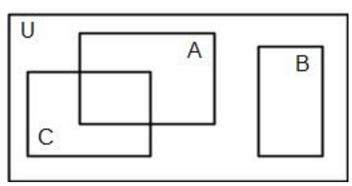
4. Let universe set $U = \{1, 2, 3, 4, 5, 6, 7\}$, and sets $A = \{1, 2\}$, $B = \{2, 3, 4\}$, $C = \{4, 5, 6, 7\}$. Find

   - $A \times C$
   - $A \bigcap B \bigcap C$
   - $|C|$
   - $\overline{B}$
   - $A - B$
   - $(A \bigcup C) \bigcap B$

5. For the given sets (see the Venn Diagramm) show on the Diagramm



a) $A \bigcap C \bigcap \overline{B}$



b) $\overline{A \bigcup B} \bigcap C$

6. Use set identities to prove the following identity:

   $\overline{A \cup \overline{B}} = \overline{A} \cap B$

Functions

1. Determine whether the function $f(x) = x^5 + 1$, where $f(x) : Z \to Z$ is

   a) one-to-one?

   b) onto?

   c) bijection?

2. Determine the domain, the codomain/target, and the range of the function $f(x) = x^2 + 3$

3. Let $f(x) = 2x$ and $g(x) = 3x + 5$, are functions from $\mathbf{R}$ to $\mathbf{R}$.

   a) Find $f \circ g$ and $g \circ f$

   b) Find $(fg)(x)$

   c) Find $(f + g)(x)$

4. Is function $g(x) = x^3 - 4$ invertible? If yes, explain why and find its inverse function. If no, provide explanations also.

Algorithms

1. Give an algorithm, using pseudocode, that takes a list of n distinct integers as input and finds the location of the largest odd integer in the list. If there is no such integer, the algorithm should return 0.

2. Present the algorithm, using a pseudocode, of finding the largest integer in an unordered sequence of n integers.

   Don't forget to describe the input and output for your algorithm.

3. Take a look at the following algorithm:

```
procedure it(n:positive integer)
sum := 0
For i:= 1 to n
    sum := sum + i*i
End-for
Return(sum)
```

   (a) How many multiplication operations will be done (an expression with `n`)?

   (b) If $n = 3$, what value will be returned?

4. Given the algorithm:

```
procedure thing(a_1,a_2,a_3,...a_n:integers)
sum1 := 0
sum2 := 0
For i := 1 to n
   If (a_i > 0), sum1 := sum1 + a_i
   If (a_i < 0), sum2 := sum2 + a_i
End-for
Return(sum1,sum2)
```

   For the set of values {1,5,-2,-9,2,5,-7} as input for the above algorithm, what are values of `sum1` and `sum2` that will be returned?

5. Consider the algorithm:

```
procedure foo(n:integer)
If n > 10, print('A')
If (n <= 10) and (n > -10), print('B')
Else print('C')
```

a) What will be printed if the procedure `foo` is run on `n=4`?

b) What will be printed if the procedure `foo` is run on `n=-4`?

c) What will be printed if the procedure `foo` is run on `n=24`?

6. Use Linear search to find 13 in the following list: $1, 7, 2, 3, 6, 8, 13, 4, 89$
   How many comparisons will be performed?

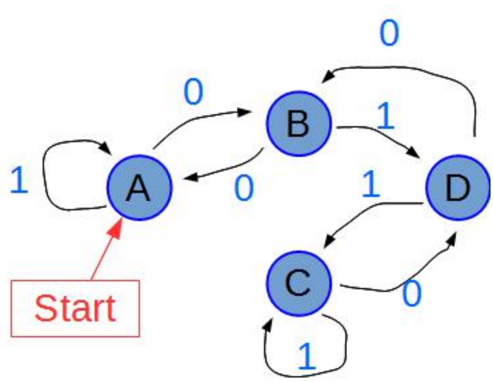
7. Can binary search be used if it gets the following list as input: $1, 7, 2, 3, 6, 8, 13, 4, 89$?

8. Use binary search to find 14 in the following list: $1, 6, 8, 9, 13, 14, 16, 22, 36, 38$
   Show all the splits and all the middle elements.

9. If binary search is used to find 10 in the following list: $9, 10, 14, 16, 22, 36, 56, 59, 61$
   How many splits will be performed before the element is found?

10. Use **bubble sort** algorithm to sort the list $5, 2, 4, 1, 3$
    (show all the passes, with interchanges, see our lecture slides)
    How many interchanges will be performed during the first pass?

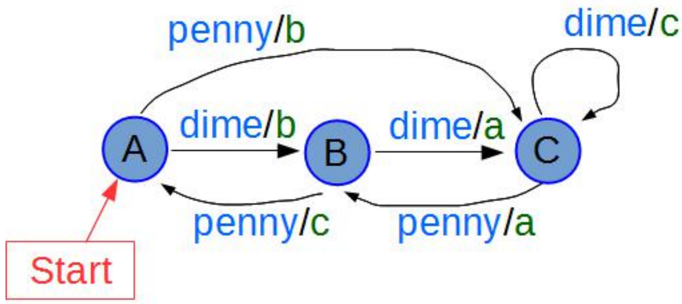
11. For the following FSM



1) What is the current state after the FSM has processed the input sequence `0 1 0 1 1`?

2) What input sequence required to get from state A to state A and changing to at least one other state?

12. For the following FSM



1) What is the current state after the FSM has processed the input sequence `PENNY DIME PENNY DIME`?

2) What input sequence required to get from state A to state A and changing to at least one other state?

Integers

1. Find

   a) 243 **div** 13 **b)** 243 **mod** 13 **c)** (-100) **div** 23 **d)** (-100) **mod** 23

2. State True or False, explain why.

   $12 \equiv 54(\mathbf{mod}\ 7)$

3. Give prime factorization of 231, if possible.

4. Check if 421, 387, and 157 are prime numbers

   (use Theorem 6.4.1: Small factors from zyBooks)

5. Find $gcd(231, 42)$ and $lcm(231, 42)$

6. Check if the given pairs are relatively prime

   a) 42, 154 **b)** 396, 175

7. Find the expansion base 5 of 187.

8. Find the expansion base 16 of 187.

9. Find the binary of 49.

10. Compute the decimal representation of $(012112)_3$

11. Compute the decimal representation of $(77501)_8$

12. Compute the decimal representation of $(A501)_{12}$

    digit A represents 10, and digit B represents 11.

13. Compute 5^6 **mod** 7 without use of calculator

14. What is the value of $26 + 28$ in Z_{30}?

15. What is the value of 16×20 in Z_{30}?

16. Check which integers are multiplicative inverses of 5 $\texttt{mod}$ 59.

    a) 12 **b)** 46 **c)** 71

17. Compute $(351^8 + 86)$ **mod** 10 without use of calculator

18. Compute $[(43 \texttt{ mod } 10) + (76 \texttt{ mod } 10)] \texttt{ mod } 10$

19. Consider the following ***linear congruential generator*** of pseudo-random numbers:

    $x_n = (5x_{n-1} + 7) \texttt{ mod } 12$, with seed $x_0 = 4$

    What sequence of pseudo-random numbers does it generate?

20. Consider a simple ***cryptosystem*** in which the set of all possible *plaintexts* m come from Z_N for some integer N. Alice and Bob share a secret number $k \in Z_N$. The security of their encryption scheme rests on the assumption that no one besides them knows the number k. To encrypt a *plaintext* $m \in Z_N$, Alice computes:

    $c = (m + k) \texttt{ mod } N$ (encryption)

    Alice sends the *ciphertext* c to Bob. When Bob receives the *cyphertext* c, he decrypts c as follows:

    $m = (c - k) \texttt{ mod } N$ (decryption)

    Assume that $N = 650$, and Alice and Bob agreed on key $k = 127$

    a) Alice wants to send the *plaintext* message $m = 345$ to Bob.

    What will be the encrypted message, *cyphertext* c?

    b) Bob received *cyphertext* $c = 534$ from Alice.

    What will be the *plaintext* message m after he decrypts the *cyphertext*?

    c) Suppose that Eve somehow found out that $N = 650$ and also managed to learn that message $m = 555$ corresponds to $c = 32$. Can she infer/deduce the value for key k? Show how.

Counting

1. At a bakery, there are three shapes of bread that can be made: rolls, bowl, loaf, and extra large loaf. Each bread can be of type: Ciabatta, Country, Whole Grain, Classic White, and Focaccia. Each bread can be sliced or not. How many different choices of bread are available?

2. How many bit-strings of length 16 are there, that start with 0 or end with 11?

3. List all 3-permutations and 3-combinations (3-subsets) of the set $S = \{1, 2, 3, 4, 5\}$.

4. Find the values of $P(16, 5)$ and $C(26, 4) = \binom{26}{4}$.

5. How many bit-strings of length 14 contain
   (a) five 0's in the beginning?
   (b) at least four 1's?
   (c) number of 1's and the number of 0's are the same?

6. In how many ways can the letters of the word 'COLUMN' be arranged?

7. In how many ways can the letters of the word 'ABRAKADABRA' be arranged?

8. There are 10 men and 8 women in a room. In how many ways we can choose 4 men and 4 women from the room?

9. How many integers from 1 to 50 are only multiples of 2 or 3?

10. In a group of 50 students 24 like cold drinks and 36 like hot drinks and each student likes at least one of the two drinks. How many like both coffee and tea?

11. How many passwords of length 5, that contain only lower case letters and four special symbols ‘, −, +, _ can be made?