

Chapter 1 cheat sheet

Logical operators, their truth tables, laws:

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of Logical Operators: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \oplus$

Laws of Propositional logic:

$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's Laws (1)	$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws (5)
$\neg(\neg p) \equiv p$	Double negation law (2)	$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws (6)
$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative laws (3)	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws (7)
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws (4)	$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws (8)
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws (9)		
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws (10)		

$p \rightarrow q \equiv \neg p \vee q$ (11)	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (12)
$p \rightarrow q \equiv \neg q \rightarrow \neg p$ (13)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ (14)
$p \vee q \equiv \neg p \rightarrow q$ (15)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (16)
$p \wedge q \equiv \neg(p \rightarrow \neg q)$ (17)	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ (18)
$\neg(p \rightarrow q) \equiv p \wedge \neg q$ (19)	
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ (20)	
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$ (21)	
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$ (22)	
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ (23)	

\forall – universal quantifier \exists – existential quantifier $\exists!$ – uniqueness quantifier $\rightarrow \wedge \vee \neg$

Precedence of Quantifiers and Logical Operators: $\forall, \neg, \wedge, \vee, \rightarrow, \leftrightarrow$

$\exists, \exists!$

Quantifiers' precedence: from left to the right

Quantifiers with restricted domain:

$\forall x < 0 (x^2 > 0)$ can be re-written with *implication* $\forall x (x < 0 \rightarrow x^2 > 0)$

$\exists x > 0 (x^2 = 4)$ can be re-written with *conjunction* $\exists x (x > 0 \wedge x^2 = 4)$

Negating Quantified expressions (De Morgan's Laws):

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

Nested quantifiers:

Statement	When True?	When False?
$\forall y \forall x P(x,y)$ $\forall x \forall y P(x,y)$	P(x,y) is true for any pair x, y	there is a pair x, y for which P(x,y) is False
$\forall x \exists y P(x,y)$	For any x, there is a y, for which P(x,y) is true	There is an x, for which P(x,y) is false for any y
$\exists x \forall y P(x,y)$	There exists an x, for which P(x,y) is true for any y	There is no x, such that P(x,y) is true for any y
$\exists x \exists y P(x,y)$ $\exists y \exists x P(x,y)$	There is a pair x, y for which P(x,y) is true	P(x,y) is false for every pair x, y

Rules of inference:

Rule of inference	Name		
$\frac{p \quad p \rightarrow q}{\therefore q}$	Modus ponens	$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	Modus tollens
$\frac{p}{\therefore p \vee q}$	Addition	$\frac{p \wedge q}{\therefore p}$	Simplification
$\frac{p \quad q}{\therefore p \wedge q}$	Conjunction	$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	Hypothetical syllogism
$\frac{p \vee q \quad \neg p}{\therefore q}$	Disjunctive syllogism	$\frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$	Resolution