Chapter 1 cheat sheet

Logical operators, their truth tables, laws:

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of Logical Operators: $\neg, \wedge, \vee, \to, \leftrightarrow$

Laws of Propositional logic:

$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's Laws (1)	$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws (5)
$\neg(\neg p) \equiv p$	Double negation law (2)	$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws (6)
$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative laws (3)	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws (7)
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws (4)	$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws (8)
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$		Associative laws (9)	
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$		Distributive laws (10)	

$p \to q \equiv \neg p \vee q$ (11)	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ (12)
$p \to q \equiv \neg q \to \neg p$ (13)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$ (14)
$p \vee q \equiv \neg p \to q$ (15)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (16)
$p \wedge q \equiv \neg(p \to \neg q)$ (17)	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ (18)
$\neg(p \to q) \equiv p \wedge \neg q$ (19)	
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$ (20)	
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$ (21)	
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$ (22)	
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ (23)	

$\forall$ – universal quantifier $\exists$ – existential quantifier $\exists!$ – uniqueness quantifier $\rightarrow \wedge \vee \neg$

Precedence of Quantifiers and Logical Operators: $\forall$ $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
$\exists, \exists!$

Quantifiers' precedence: from left to the right

Quantifiers with restricted domain:

$\forall x<0\ (x^2>0)$ can be re-written with *implication* $\forall x\ (x<0 \rightarrow x^2>0)$

$\exists x>0\ (x^2=4)$ can be re-written with *conjunction* $\exists x\ (x>0 \wedge x^2=4)$

Negating Quantified expressions (De Morgan's Laws):

$\neg\exists x P(x) \equiv \forall x \neg P(x)$ $\neg\forall x P(x) \equiv \exists x \neg P(x)$

Nested quantifiers:

Statement	**When True?**	**When False?**
$\forall y\ \forall x\ P(x,y)$ $\forall x\ \forall y\ P(x,y)$	P(x,y) is true for any pair x, y	there is a pair x, y for which P(x,y) is False
$\forall x\ \exists y\ P(x,y)$	For any x, there is a y, for which P(x,y) s true	There is an x, for which P(x,y) is false for any y
$\exists x\ \forall y\ P(x,y)$	There exists an x, for which P(x,y) is true for any y	There is no x, such that P(x,y) is true for any y
$\exists x\ \exists y\ P(x,y)$ $\exists y\ \exists x\ P(x,y)$	There is a pair x, y for which P(x,y) is true	P(x,y) is false for every pair x, y

Rules of inference:

Rule of inference	Name		
p $p \rightarrow q$ ——— $\therefore q$	Modus ponens	$\neg q$ $p \rightarrow q$ ——— $\therefore \neg p$	Modus tollens
p ——— $\therefore p \vee q$	Addition	$p \wedge q$ ——— $\therefore p$	Simplification
p q ——— $\therefore p \wedge q$	Conjunction	$p \rightarrow q$ $q \rightarrow r$ ——— $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ ——— $\therefore q$	Disjunctive syllogism	$p \vee q$ $\neg p \vee r$ ——— $\therefore q \vee r$	Resolution