

Let's find $\gcd(e, j) = \gcd(197, 352)$ using Extended Euclid's Algorithm:

$$\gcd(197, 352) =$$

$$x = 197 \text{ (divisor)}$$

$$y = 352 \text{ (dividend)}$$

$$d = 352 \text{ div } 197 = 1$$

$$r = 352 - 1 * 197 = 155$$

$$y = x = 197$$

$$x = r = 155$$

$$= \gcd(155, 197) =$$

$$d = 197 \text{ div } 155 = 1$$

$$r = 197 - 1 * 155 = 42$$

$$y = x = 155$$

$$x = r = 42$$

$$= \gcd(42, 155) =$$

$$d = 155 \text{ div } 42 = 3$$

$$r = 155 - 3 * 42 = 29$$

$$y = x = 42$$

$$x = r = 29$$

$$= \gcd(29, 42) =$$

$$d = 42 \text{ div } 29 = 1$$

$$r = 42 - 1 * 29 = 13$$

$$y = x = 29$$

$$x = r = 13$$

$$= \gcd(13, 29) =$$

$$d = 29 \text{ div } 13 = 2$$

$$r = 29 - 2 * 13 = 3$$

$$y = x = 13$$

$$x = r = 3$$

$$= \gcd(3, 13) =$$

$$d = 13 \text{ div } 3 = 4$$

$$r = 13 - 4 * 3 = 1$$

$$y = x = 3$$

$$x = r = 1$$

$$= \gcd(1, 3) =$$

$$1 = 109 * 197 - 61 * 352$$

$$1 = 48 * 197 - 61 * 352 + 61 * 197$$

$$1 = 48 * 197 - 61 * (352 - 1 * 197)$$

$$1 = 48 * 197 - 61 * 155$$

$$1 = 48 * (197 - 1 * 155) - 13 * 155$$

$$1 = 48 * 42 - 13 * 155$$

$$1 = 9 * 42 - 13 * 155 + 39 * 42$$

$$1 = 9 * 42 - 13 * (155 - 3 * 42)$$

$$1 = 9 * 42 - 13 * 29$$

$$1 = 9 * (42 - 1 * 29) - 4 * 29$$

$$1 = 9 * 13 - 4 * 29$$

$$1 = 13 - 4 * 29 + 8 * 13$$

$$1 = 13 - 4 * (29 - 2 * 13)$$

$$1 = 13 - 4 * 3$$

Working from the bottom up:



$$\begin{aligned}d &= 3 \operatorname{div} 1 = 3 \\r &= 3 - 3 * 1 = 0 \\y &= x = 1 \\x &= r = 0\end{aligned}$$

$$= 1$$

$$\operatorname{gcd}(197, 352) = 1 = 109 * 197 - 61 * 352 \quad \text{matching it with } \operatorname{gcd}(x, y) = s * x + t * y$$

$$s = 109, t = 61$$

multiplicative inverse of $x \bmod y$ is $s \bmod y$.

Therefore, multiplicative inverse of $197 \bmod 352$ is $109 \bmod 352 = \mathbf{109}$