Chapter 7. Intro to Counting

7.1 Sum and product rules
7.2 The bijection rule (self-study)
7.3 The generalized product rule
7.4 Counting permutations
7.5 Counting subsets
7.6 Subset and permutation examples

7.1 Sum and product rules

Counting problems arise throughout mathematics and computer science.

Example: consider the following requirements for a password for an e-mail account:

- should consist of six to eight characters.
- each of these characters must be a digit or a letter of the alphabet (lower case or upper case)
- each password must contain at least one digit and at least one character.

How many passwords are there?

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• each of these characters must be a digit or a letter of the alphabet (lower case or upper case)
• each password must contain at least one digit and at least one character.

How many passwords are there? we will get back to it later

The product rule:
(*applies when a procedure is made up of separate tasks*)
Suppose that a procedure can be broken down into sequence of two tasks.

If there are n_1 ways to do the first task, and n_2 ways to do the second task (for each of those n_1 ways), then there are $n_1 \cdot n_2$ ways to do the procedure.

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Example 1: There are 20 students in our CSI30 class. Each student has a pair of hands. How many hands do we have in our CSI30 class?

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Example 1: There are 20 students in our CSI30 class. Each student has a pair of hands. How many hands do we have in our CSI30 class?

20 · 2 = 40 hands

Example 2: how many difference license plates are available if each plate contains a sequence of three letters followed by two digits?

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<u>Solution</u>:

_ _ _ _ _

letters digits

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Solution:

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letters digits

26 26 26 10 10

26 · 26 · 26 · 10 · 10 = 1 757 600

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Example 3: Let's talk about bit-strings (binary strings).
Alphabet (digits): 0,1
a) How many bit-strings of length 7 is there?

b) How many binary strings of length 10 is there that start and end with digit 1?

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Alphabet (digits): 0,1
a) How many bit-strings of length 7 is there? 2^7

places:	_	_	_	_	_	_	_	
options:	2	2	2	2	2	2	2	Total: 2^7

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places:	_	_	_	_	_	_	_	
options:	2	2	2	2	2	2	2	Total: 2^7

b) How many binary strings of length 10 is there that start and end with digit 1? 2^8

Places:	<u>1</u>	_	_	_	_	_	_	_	_	<u>1</u>	
Options:	1	2	2	2	2	2	2	2	2	1	Total: 2^8

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The sum rule:
If a task can be done either in one of n_1 ways or in one of n_2 ways (different ways), then there are n_1+n_2 ways to do the task.

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A student can choose a computer project from one of two lists. Each list contains of *12* and *19* possible projects, respectively (all projects are different). How many possible projects are there to choose from?

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A student can choose a computer project from one of two lists. Each list contains of *12* and *19* possible projects, respectively (all projects are different). How many possible projects are there to choose from?

Solution: $19 + 12 = 31$

Example 5: How many additions will be done in the following algorithm?

procedure *this*($n_1, n_2, n_3, \ldots, n_k$: positive integers)
$s = 0$
for $i := 1$ **to** n_1
    $s := s+1$
for $i := 1$ **to** n_2
    $s: = s+2$
for $i := 1$ **to** n_3
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for $i := 1$ **to** n_k
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Solution:
in the first loop: n_1 additions will be performed (number of iterations)

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    $s := s+k$

<u>Solution</u>:
in the first loop: n_1 additions will be performed (number of iterations)
in the second loop: n_2

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procedure *this*($n_1,n_2,n_3,...,n_k$: positive integers)
$s = 0$
for $i := 1$ **to** n_1
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    $s: = s+2$
for $i := 1$ **to** n_3
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....
for $i := 1$ **to** n_k
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<u>Solution</u>:
in the first loop: n_1 additions will be performed (number of iterations)
in the second loop: n_2
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in the last, k^{th} loop: n_k

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Solution:
in the first loop: n_1 additions will be performed (number of iterations)
in the second loop: n_2
...
in the last, k^{th} loop: n_k
Therefore, there will be $n_1 + n_2 + n_3 + \ldots + n_k$ additions

7.3 The generalized product rule

Generalized product rule:
Consider a set S of sequences of k items. Suppose there are:

- n_1 choices for the first item
- for every possible choice for the first item, there are n_2 choices for the second item
- for every possible choice for the first and second items, there are n_3 choices for the third item.

...
for every possible choice for the first k-1 items, there are n_k choices for the k^{th} item

Then $|S| = n_1 \times n_2 \times \ldots \times n_k$

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Solution:
choices: color sex size

$$12 \cdot 2 \cdot 3 = 72$$

The generalized sum rule

Example:

Assume that $\mathbf{A_1, A_2, ..., A_n}$ are disjoint sets. How many elements are there in their union $\mathbf{A_1} \cup \mathbf{A_2} \cup \mathbf{...} \cup \mathbf{A_n}$? (sum rule)

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<u>Answer</u>:
$| \mathbf{A_1} \cup \mathbf{A_2} \cup \ldots \cup \mathbf{A_n}| = |\mathbf{A_1}| + |\mathbf{A_2}| + \ldots + |\mathbf{A_n}|$

7.4 Counting permutations

Many counting problems can be solved by finding the number of ways to arrange a specified number of distinct elements of a set of a particular size (where the order of the elements matters).

Example: In how many ways can we select three students from a group of seven students to stand in line for a picture?

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students: “A”,”B”,”C”,”D”,”E”,”F”,”G”
possible arrangements: “A”,”B”,”C”;
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A **permutation** of a set of distinct objects is an ordered arrangement of these objects.

An ordered arrangement of *r* elements of a set (that has more than r elements) is called an ***r*-permutation**.

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The generalized product rule is often used in counting permutations.

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places: ____ ____ ____
options: 7 6 5
number of arrangements: $7 \times 6 \times 5$ = 210 ways

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7 6 5 4 3 2 1 = 7! = 5040 ways

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students: "A","B","C","D","E","F","G"
possible arrangements: "A","B","C";
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no repetitions, ordered

places: ___ ___ ___
options: 7 6 5

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In how many ways can we arrange all seven of these students in a line for a picture?

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The number of *r*-permutations from a set with *n* elements, where $r, n \in \mathbf{Z}^+$, with $r \leq n$.

$$P(n, r) = \frac{n!}{(n-r)!} = \frac{n(n-1)\ldots(n-r)(n-r-1)\ldots(1)}{(n-r)(n-r-1)\ldots(1)} = n(n-1)(n-2)\ldots(n-r+1)$$

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Example:
List all the permutations of set {1,2,3,4}

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1,2,3,4; 1,2,4,3 ; 1,3,2,4; 1,3,4,2; 1,4,3,2; 1,4,2,3

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3,1,2,4;	3,1,4,2;	3,2,4,1;	3,2,1,4;	3,4,1,2;	2,4,2,1
4,1,2,3;	4,1,3,2;	4,2,3,1;	4,2,1,3;	4,3,1,2;	4,3,2,1

There are *24* permutations of a set with four elements (objects)

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3,1,2,4;	3,1,4,2;	3,2,4,1;	3,2,1,4;	3,4,1,2;	2,4,2,1
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Using formula, $P(4,4) = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4\times3\times2\times1 = 24$

or $P(4,4) = 4\,(4-1)(4-2)(4-3) = 4\,(3)(2)(1) = 24$

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Example:
List all the *2*-permutations of set {a,b,c,d}

<u>Solution</u>:
a,b; a,c; a,d; b,a; b,c; b,d; c,a; c,b; c,d
d,a; d,b; d,c

There are *12* *2*-permutations of a set with four elements (objects)

P(*4,2*) = *4 (4-1)* = *4 (3)* = *12*

7.4 Counting permutations

$$P(n,r)=\frac{n!}{(n-r)!}=\frac{n(n-1)\ldots(n-r)(n-r-1)\ldots(1)}{(n-r)(n-r-1)\ldots(1)}=n(n-1)(n-2)\ldots(n-r+1)$$

Example:
There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

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<u>Solution</u>:
six candidates, permutations

$$P(6,6)=\frac{6!}{(6-6)!}=\frac{6!}{(0)!}=\frac{6!}{1}=1*2*3*4*5*6=720$$

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In how many ways can a set of two positive integers less than or equal to *100* be chosen?

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Example:
In how many ways can a set of two positive integers less than or equal to *100* be chosen?

Solution:
100 numbers: from *1* to *100*
2-permutations
Therefore, we are looking for $P(100,2) = \frac{100!}{(100-2)!} = \frac{100!}{98!} = 99*100 = 9900$

7.5 Counting subsets

Consider the following situation:
a class with 20 students who must elect three representatives to the student council. The teacher conducts a vote and reveals the names of the three students who received the most votes. He does not reveal how many votes each student received or which one received more votes than the other two. How many ways are there to select the three representatives?

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Note that the outcome of the election is a <u>set of three students</u>, not a sequence because there is no particular order imposed on the three representatives. The outcome {Joshua, Karen, Ingrid} is the same outcome as {Karen, Ingrid, Joshua}.

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List all the 2-combinations of set {a,b,c,d}

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There are *6* *2*-combinations of a set with four elements (objects)

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~~d,a~~; ~~d,b~~; ~~d,c~~

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[Denotation]
The number of *r*-combinations of a set with n elements is denoted by *C(n,r)* and is called **binomial coefficient**.

Another denotation: $\binom{n}{r}$

If $n,\ r \in \mathbb{Z}$, with $0 \le r \le n$, then there are $\boxed{C(n,r) = \dfrac{n!}{r!(n-r)!}}$ *r*-combinations of a set with *n* distinct elements.

$$\boxed{C(n,r)=\frac{n!}{r!(n-r)!}}$$

Example:

How many ways are there to select 6 cards from a standard deck of 52 cards?

$$\boxed{C(n, r) = \frac{n!}{r!(n-r)!}}$$

Example:

How many ways are there to select 6 cards from a standard deck of 52 cards?

Solution:

usually the order of cards dealt is not important.

Therefore, we are talking about 6-combinations out of 52 cards.

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$$C(52,6) = \frac{52!}{6!(52-6)!} = \frac{52!}{6!(46)!} = \frac{47 * \overset{1\,8}{\cancel{48}} * 49 * \overset{10}{\cancel{50}} * \overset{17}{\cancel{51}} * 52}{1 * \cancel{2} * \underset{1}{\cancel{3}} * \cancel{4} * \underset{1}{\cancel{5}} * \underset{1}{\cancel{6}}} = 20,358,520$$

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Example:
How many bit strings of length *12* contain exactly three *1*s?

7.5 Counting subsets

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Example:
How many ways are there to select 6 cards from a standard deck of 52 cards?

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usually the order of cards dealt is not important.
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Example:
How many bit strings of length *12* contain exactly three *1*s?

<u>Solution</u>:
bit strings contain *0*s and *1*s.
We need the bit strings that have three *1*s and nine *0*s.

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

Example:
How many ways are there to select 6 cards from a standard deck of 52 cards?

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usually the order of cards dealt is not important.
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$$C(52,6) = \frac{52!}{6!(52-6)!}=\frac{52!}{6!(46)!}=\frac{47*\overset{1\,8}{\cancel{48}}*49*\overset{10}{\cancel{50}}*\overset{17}{\cancel{51}}*52}{1*\cancel{2}*\underset{1}{\cancel{3}}*\cancel{4}*\underset{1}{\cancel{5}}*\underset{1}{\cancel{6}}}=\boxed{20,358,520}$$

Example:
How many bit strings of length *12* contain exactly three *1*s?

<u>Solution</u>:
bit strings contain *0*s and *1*s.
We need the bit strings that have three *1*s and nine *0*s.

<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>
1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}

If order of *1*s important?

$$\boxed{C(n,r) = \frac{n!}{r!(n-r)!}}$$

Example:
How many ways are there to select 6 cards from a standard deck of 52 cards?

Solution:
usually the order of cards dealt is not important.
Therefore, we are talking about 6-combinations out of 52 cards.

$$C(52,6) = \frac{52!}{6!(52-6)!} = \frac{52!}{6!(46)!} = \frac{47*\overset{1\ \cancel{8}}{\cancel{48}}*49*\overset{10}{\cancel{50}}*\overset{17}{\cancel{51}}*52}{1*\cancel{2}*\underset{1}{\cancel{3}}*\cancel{4}*\underset{1}{\cancel{5}}*\underset{1}{\cancel{6}}} = 20{,}358{,}520$$

Example:
How many bit strings of length *12* contain exactly three *1*s?

Solution:
bit strings contain *0*s and *1*s.
We need the bit strings that have three *1*s and nine *0*s.

1	1	1	0	0	0	0	0	0	0
1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th

If order of *1*s important? No.
Therefore, we are looking for 3-combination of 12 objects.

$$C(12,3) = \frac{12!}{3!(12-3)!} = \frac{12!}{3!(9)!} = \frac{10*11*12}{1*2*3} = 220$$