

Chapter 7. Intro to Counting

- 7.1 Sum and product rules
- 7.2 The bijection rule (self-study)
- 7.3 The generalized product rule
- 7.4 Counting permutations
- 7.5 Counting subsets
- 7.6 Subset and permutation examples

7.1 Sum and product rules

Counting problems arise throughout mathematics and computer science.

Example: consider the following requirements for a password for an e-mail account:

- should consist of six to eight characters.
- each of these characters must be a digit or a letter of the alphabet (lower case or upper case)
- each password must contain at least one digit and at least one character.

How many passwords are there?

7.1 Sum and product rules

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How many passwords are there? [we will get back to it later](#)

7.1 Sum and product rules

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The product rule:

(applies when a procedure is made up of separate tasks)

Suppose that a procedure can be broken down into sequence of two tasks.

If there are n_1 ways to do the first task, and n_2 ways to do the second task (for each of those n_1 ways), then there are $n_1 \cdot n_2$ ways to do the procedure.

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Example 1: There are 20 students in our CSI30 class. Each student has a pair of hands. How many hands do we have in our CSI30 class?

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$$20 \cdot 2 = 40 \text{ hands}$$

7.1 Sum and product rules

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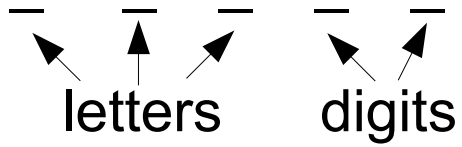
Example 2: how many different license plates are available if each plate contains a sequence of three letters followed by two digits?

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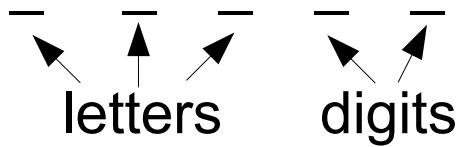
Solution:



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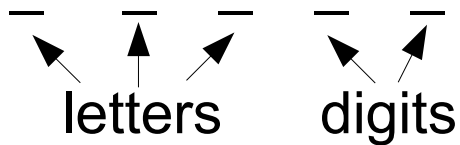
26 26 26 10 10

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 = 1\,757\,600$$

7.1 Sum and product rules

Example 2: how many difference license plates are available if each plate contains a sequence of three letters followed by two digits?

Solution:



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Example 3: Let's talk about bit-strings (binary strings).

Alphabet (digits): 0, 1

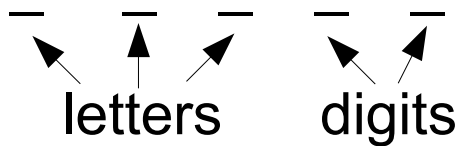
a) How many bit-strings of length 7 is there?

b) How many binary strings of length 10 is there that start and end with digit 1?

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a) How many bit-strings of length 7 is there? 2^7

places:

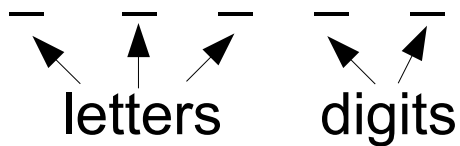
options: $\bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2}$ Total: 2^7

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b) How many binary strings of length 10 is there that start and end with digit 1? 2^8

Places: $\frac{1}{1} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \bar{2} \ \frac{1}{1}$ Total: 2^8

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The sum rule:

If a task can be done either in one of n_1 ways or in one of n_2 ways (different ways), then there are n_1+n_2 ways to do the task.

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Example 4:

A student can choose a computer project from one of two lists. Each list contains of 12 and 19 possible projects, respectively (all projects are different). How many possible projects are there to choose from?

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Solution: $19 + 12 = 31$

7.1 Sum and product rules

Example 5: How many additions will be done in the following algorithm?

procedure *this*($n_1, n_2, n_3, \dots, n_k$: positive integers)

$s = 0$

for $i := 1$ **to** n_1

$s := s + 1$

for $i := 1$ **to** n_2

$s := s + 2$

for $i := 1$ **to** n_3

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Solution:

in the first loop: n_1 additions will be performed (number of iterations)

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Solution:

in the first loop: n_1 additions will be performed (number of iterations)

in the second loop: n_2

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Therefore, there will be $n_1 + n_2 + n_3 + \dots + n_k$ additions

7.3 *The generalized product rule*

Generalized product rule:

Consider a set S of sequences of k items. Suppose there are:

- n_1 choices for the first item
- for every possible choice for the first item, there are n_2 choices for the second item
- for every possible choice for the first and second items, there are n_3 choices for the third item.
- ...
- for every possible choice for the first $k-1$ items, there are n_k choices for the k^{th} item

Then $|S| = n_1 \times n_2 \times \dots \times n_k$

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Example:

A particular brand of short comes in 12 colors, has a male version and female version, and comes in three sizes for each sex. How many different types of this shirt are made?

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Solution:

choices: color sex size
 12 · 2 · 3 = 72

The generalized sum rule

Example:

Assume that $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$ are disjoint sets. How many elements are there in their union $\mathbf{A}_1 \cup \mathbf{A}_2 \cup \dots \cup \mathbf{A}_n$? (sum rule)

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Example:

Assume that $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$ are disjoint sets. How many elements are there in their union $\mathbf{A}_1 \cup \mathbf{A}_2 \cup \dots \cup \mathbf{A}_n$? (sum rule)

Answer:

$$|\mathbf{A}_1 \cup \mathbf{A}_2 \cup \dots \cup \mathbf{A}_n| = |\mathbf{A}_1| + |\mathbf{A}_2| + \dots + |\mathbf{A}_n|$$

7.4 Counting permutations

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Many counting problems can be solved by finding the number of ways to arrange a specified number of distinct elements of a set of a particular size (where the order of the elements matters).

Example: In how many ways can we select three students from a group of seven students to stand in line for a picture?

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Example: In how many ways can we select three students from a group of seven students to stand in line for a picture?

students: "A", "B", "C", "D", "E", "F", "G"

possible arrangements: "A", "B", "C";

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A **permutation** of a set of distinct objects is an ordered arrangement of these objects.

An ordered arrangement of r elements of a set (that has more than r elements) is called an **r -permutation**.

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An ordered arrangement of r elements of a set (that has more than r elements) is called an **r -permutation**.

The generalized product rule is often used in counting permutations.

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$\frac{7}{7}$

$\frac{6}{6}$

$\frac{5}{5}$

number of arrangements: $7 \times 6 \times 5 = 210$ ways

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$\overline{7}$

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In how many ways can we arrange all seven of these students in a line for a picture?

$$\overline{7} \quad \overline{6} \quad \overline{5} \quad \overline{4} \quad \overline{3} \quad \overline{2} \quad \overline{1} = 7! = 5040 \text{ ways} \quad 32$$

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number of arrangements: $7 \times 6 \times 5 = 210$ ways

no repetitions,
ordered

In how many ways can we arrange all seven of these students in a line for a picture?

$$\frac{7}{7} \frac{6}{6} \frac{5}{5} \frac{4}{4} \frac{3}{3} \frac{2}{2} \frac{1}{1} = 7! = 5040 \text{ ways}$$

7.4 Counting permutations

The number of r -permutations from a set with n elements, where $r, n \in \mathbf{Z}^+$, with $r \leq n$.

$$P(n, r) = \frac{n!}{(n-r)!} = \frac{n(n-1)\dots(n-r)(n-r-1)\dots(1)}{(n-r)(n-r-1)\dots(1)} = n(n-1)(n-2)\dots(n-r+1)$$

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1,2,3,4; 1,2,4,3; 1,3,2,4; 1,3,4,2; 1,4,3,2; 1,4,2,3

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2,1,3,4; 2,1,4,3; 2,3,4,1; 2,3,1,4; 2,4,1,3; 2,4,3,1
3,1,2,4; 3,1,4,2; 3,2,4,1; 3,2,1,4; 3,4,1,2; 2,4,2,1
4,1,2,3; 4,1,3,2; 4,2,3,1; 4,2,1,3; 4,3,1,2; 4,3,2,1

There are 24 permutations of a set with four elements (objects)

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3,1,2,4; 3,1,4,2; 3,2,4,1; 3,2,1,4; 3,4,1,2; 2,4,2,1
4,1,2,3; 4,1,3,2; 4,2,3,1; 4,2,1,3; 4,3,1,2; 4,3,2,1

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Using formula, $P(4,4) = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4 \times 3 \times 2 \times 1 = 24$

or $P(4,4) = 4 (4-1)(4-2)(4-3) = 4 (3)(2)(1) = 24$

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Example:

List all the 2-permutations of set $\{a,b,c,d\}$

Solution:

a,b; a,c; a,d; b,a; b,c; b,d; c,a; c,b; c,d
d,a; d,b; d,c

There are **12** 2-permutations of a set with four elements (objects)

$$P(4,2) = 4 (4-1) = 4 (3) = 12$$

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Example:

There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

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six candidates, permutations

$$P(6, 6) = \frac{6!}{(6-6)!} = \frac{6!}{(0)!} = \frac{6!}{1} = 1 * 2 * 3 * 4 * 5 * 6 = 720$$

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Example:

In how many ways can a set of two positive integers less than or equal to 100 be chosen?

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Example:

In how many ways can a set of two positive integers less than or equal to 100 be chosen?

Solution:

100 numbers: from 1 to 100

2-permutations

Therefore, we are looking for $P(100, 2) = \frac{100!}{(100-2)!} = \frac{100!}{98!} = 99 * 100 = 9900$

7.5 Counting subsets

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Consider the following situation:

a class with 20 students who must elect three representatives to the student council. The teacher conducts a vote and reveals the names of the three students who received the most votes. He does not reveal how many votes each student received or which one received more votes than the other two. How many ways are there to select the three representatives?

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Note that the outcome of the election is a set of three students, not a sequence because there is no particular order imposed on the three representatives. The outcome {Joshua, Karen, Ingrid} is the same outcome as {Karen, Ingrid, Joshua}.

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Solution:

$a,b;$ $a,c;$ $a,d;$ $b,a;$ $b,c;$ $b,d;$ $c,a;$ $c,b;$ c,d
 $d,a;$ $d,b;$ d,c

7.5 Counting subsets

An **r -combination** or **r -subset** of elements of a set (with n elements) is an unordered selection of r elements from the set, i.e. it is just a subset of the original set.

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There are **6** 2-combinations of a set with four elements (objects)

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[Denotation]

The number of r -combinations of a set with n elements is denoted by $C(n,r)$ and is called **binomial coefficient**.

Another denotation: $\binom{n}{r}$

If $n, r \in \mathbb{Z}$, with $0 \leq r \leq n$, then there are

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

r -combinations of a set with n distinct elements.

7.5 Counting subsets

CSI30

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Example:

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$$C(52, 6) = \frac{52!}{6!(52-6)!} = \frac{52!}{6!(46)!} = \frac{47 \overset{1}{\cancel{*}} \overset{8}{\cancel{48}} \overset{10}{\cancel{*}} \overset{17}{\cancel{49}} \overset{10}{\cancel{*}} \overset{17}{\cancel{50}} \overset{17}{\cancel{*}} \overset{17}{\cancel{51}} \overset{17}{\cancel{*}} 52}{1 \underset{1}{\cancel{*}} 2 \underset{1}{\cancel{*}} 3 \underset{1}{\cancel{*}} 4 \underset{1}{\cancel{*}} 5 \underset{1}{\cancel{*}} 6} = 20,358,520$$

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Example:

How many bit strings of length 12 contain exactly three 1s?

Solution:

bit strings contain 0s and 1s.

We need the bit strings that have three 1s and nine 0s.

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$$\begin{array}{cccccccccc} \underline{1} & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ 1^{\text{st}} & 2^{\text{nd}} & 3^{\text{rd}} & 4^{\text{th}} & 5^{\text{th}} & 6^{\text{th}} & 7^{\text{th}} & 8^{\text{th}} & 9^{\text{th}} & 10^{\text{th}} \end{array}$$

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1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th

If order of 1s important? No.

Therefore, we are looking for 3-combination of 12 objects.

$$C(12, 3) = \frac{12!}{3!(12-3)!} = \frac{12!}{3!(9)!} = \frac{10 * 11 * 12}{1 * 2 * 3} = 220$$